Replacing ‘Horn Scales’ by Act-Based Relevance Orderings
to Keep Negation and Numerals Meaningful

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Abstract: Ordered sets of words usually known as ‘pragmatic’, ‘quantitative’, or indeed ‘Horn scales’ play an crucial role in articulate meaning theories for natural language. Their order relations are widely held to be relations of entailment. Some extremely implausible consequences of this assumption will be emphasized, notably for numerals. A supportive construct, ‘metalinguistic negation’, which lacks an intelligible explanation, is also shown to be without empirical support and apt to obscure subtle and useful aspects of the data. An alternative construal of the phenomena by means of Act-Based Relevance Orderings (ABROs) within a decision-theoretic semantics (DTS) is defended. Appendix 1 recalls probability and relevance definitions, proves an illustrated theorem on ABROs and negation, and another one on ‘at least’ and conditions for ‘scalar implicature’. Appendix 2 investigates a problematic account of utterance forms which are familiar from Russell’s ‘Philosophy of Logical Atomism’. It elicits a robust relation between speech melody and the intuition, prominent also in medieval thought, that existence is a perfection. Appendix 3 observes that an existential quantification account of numeral and numeral-modifier semantics advanced by N. Kadmon and based on a modification by B. Partee of a verbal suggestion by H. Kamp, which gets ad-determinative ‘at least’ right, would have got ‘at most’ and ‘less than’ wrong. Appendix 4 contrasts model-theoretic, with epistemic and simple proof-theoretic aspects of numerical statements and relates the latter to the desiderative structure of ABROs. To clarify a point, Appendix 5 proposes a compositional semantics for exactly, as a variance-reduction operator on random variables, Appendix 6 similarly proposes basics of an act-based semantics for adsentential at least which relates the ad-determinative’s meaning to ABROs and to the phenomenology of speakers expressing themselves in adversity and consolation.

This paper consists of six sections that establish the title claim, and of an equal number of appendices, some of them long. The sections are designed to make their point with a minimum of fuss. The appendices are leisurely developments of points arising and emphasize armchair experimental data.
1. ‘Scales’. Ordered sets of words known as ‘pragmatic’ or ‘quantitative scales’ are identified by diagnostics \( X \text{ if not indeed } Y, X \text{ or even } Y \) and occasionally \( \text{not } Y \text{ and not even } X \). Instances of pairs \((X, Y)\) are \( \text{(warm, hot)} \) and \( \text{(some, all)} \), among many others, and ‘scales’\(^1\) may be relatively stable, i.e. candidates for being part of the lexicon, but also quite ad hoc to a context of use.\(^2\)

Ordered pairs of elements of Scales occur along with their diagnostics in sentences such as

(1) a. The motor is warm if not indeed \{hot / *cold\}.
   b. This cat is cool if not cold.
   c. Those drinks you served us aren’t cold, and not even cool.
   d. Some or even all cats walk.
   e. Kim or Sandy, if not indeed Kim and Sandy were present.
   f. Pacman didn’t pacify sixty realms, but only fifty.
   g. Kim lives on six if not five million dollars a year.
   h. Gizmo Golfer carded six or even five at the ninth hole.
   i. Dr Tieftank was anticipated if not indeed informed by my work.
   j. Heidi speaks Sanskrit or even Mohawk.

\(^1\)I employ scarequotes (and then substitute disquoted ‘Scale’ for comfort) because the notion of a scale has a longstanding definition in the theory of measurement. A scale is an order-homomorphism from a relational structure (e.g. an ordered set) into the real numbers. Types of scales are classified by the classes of numerical transformations under which they remain representationally invariant (see e.g. Krantz et al. 1971) The terminological nicety becomes important when we ask whether the metaphoric notion of ‘Quantity’ invoked in pragmatic explanation does admit of a coherent quantitative representation in principle. The question can usefully abstract from practical problems of assigning numerical values: an answer is an assertion about relational and algebraic properties of order and combination, which is all that a claim of numerical representability amounts to.

\(^2\)Another diagnostic, \( X \text{ but not } Y \) is less specific, being a necessary though very far from sufficient condition for Scalar paradigm membership. However, it points to a necessary condition on any adequate theory of Scales: that it come with a theory of the diagnostics used to identify its objects. There are theories of \( \text{even} \) along the late 1960s lines of Ch. Fillmore, B. Fraser, and L.R. Horn (cf. Horn 1989; and later references in Merin 1999a) and a theory along very different, argumentation-driven lines (Ducrot 1972, 1973; Merin 1999a). To prefer one of them is to incline towards the corresponding approach to Scales. Similar things hold for \( \text{but} \), of which Fogelin (1967), unlike Grice (1989), already contains a useful positive theory. It seems optimistic to assume that a non-trivial theory of putative ‘conversational implicature’ can be seriously entertained without a comprehensive and coherent theory of putative ‘conventional implicature’. Here is a quick test for one’s current favourite: to explain the star on *Kim walks but she even talks* (cf. Merin 1996/99, 1999a).
k. This is fool’s gold if not indeed real gold.

Current theorizing on connectives, quantifiers, degree particles, and other expressions of everyday language relies heavily upon such ordered sets. Assumptions made about them are the principal extrasyntactic auxiliary devices that make for a reasonably good fit between broadly logical semantics and its traditional data, spontaneous intuitions on acceptability and paraphrase.³

Scales tend to come paired as ‘positive’, e.g. (warm, ..., boiling), and ‘negative’ (cool, ..., freezing). The question of what positivity and negativity mean over and above labelling certain distributional characteristics predicated on intuitions of a distinguished, unmarked direction (see Section 2 below) is closely related to the next, and fundamental question: What intensional relation explicates or generates the observed extensional relations among Scale items?⁴

The orderings on scales—let us refer to them generically as \( R' \)—that is, the intuited orderings on lexical items, are lifted from relations \( R \) on sentences (Ducrot 1973; Horn 1972/76). The principle of ‘lifting’ is uniform and may be understood as functional abstraction. For example, warm and hot stand in a relation \( R' \) defined on predicates or properties, if sentences \( \sigma \text{ warm} \) and \( \sigma \text{ hot} \) stand in the relation \( R \) which is held to be reflected in admissibility of the diagnostic sentences.

As Gazdar (1979) pointed out in correction of Horn (1972/76), the environment \( \sigma \prec \tau \) is subject to restrictions and a straightforward lifting of \( R' \) from \( R \) tacitly takes for granted that \( \sigma X \tau \) is a ‘simple’ sentence in which \( X \) does not occur in the scope of putatively logical operators.⁵ When \( R \) is deemed to be entailment \( (Y \models X) \)—so in Horn op.cit.; so in Fogelin 1967, as a sufficient condition on sets of subalterns and extensions by intermediates in Squares of Opposition—, these ordered sets are widely known as ‘Horn scales’ [HSs].

Fogelin used his sets for a theory of scalar implicature, and thence explained (i) Aristotle’s ambiguity on possible (but not necessary) and

³The most prominent of these is the doctrine of ‘scalar implicature’ to which we turn every now and then.
⁴See Merin (1997a), a paper soon to appear in a commercial publication.
⁵There is a longstanding discussion in the literature, which kinds of environments are safe. Since negation is recognized to reverse scale ordering, it vitiates lifting, albeit in a rule-governed way. Hirschberg (1985) maintains it is the only unsafe environment, Horn (1989) proposes the larger class of downward entailing (i.e. monotonicity reversing operators), Gazdar (1979) conservatively proscribes all logical operators. In Merin (2002b), I side more closely with the conservative side.
(ii) non-lexicalization of weak negative subcontraries (e.g. not all). So did Horn (op.cit., 1973; cp. Horn 1989, 2001). Fogelin argued for orderings by strength of evidential warrant—let us dub his orderings ‘warrant scales’ [WSs]—but in practice this quickly, and avowedly, boiled down to ordering by entailment, subject to further clarification. Fogelin’s scales were vertical sides of Squares of Oppositions. They included quantifiers (with universals having their presuppositional reading), but also extralogical pairs of positives (good, not bad) and negatives (bad, not good). Clearly, not bad could be asserted when there was not warrant enough to assert good.

Ducrot (1973), by contrast, hypothesized that ordering was by strength of argumentative support for ulcerior conclusions (‘argumentative scales’ [ASs]). Thus, The motor is hot would be a stronger argument for The motor should be turned off than The motor is warm would be. Similarly, Kim and Sandy will come would be a stronger argument for (or, as the case may be, against) The conference is worth going to than Kim or Sandy will come would be. All three scholars envisaged scalar implicature engendered by non-use of a stronger item. Gazdar (1979) subsequently explicited HSs into properly predictive intelligibility and utilized an operator subclass excluding numeral and temperature terms. WSs and ASs remained pre-formal for the time being.

Ducrot (1972) also postulated a ‘metalinguistic negation’ [MLN] for presupposition rejection and (in Ducrot 1973) for a use of negation, sometimes dubbed ‘paradoxical’ [PN], which seemingly fails to reverse argu-

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6 He extended these to include an intermediate item on each vertical side. See below.

7 Ducrot (1972:134f.) is another source; Ducrot (1969) introduces échelles de signification to explain litéres, and appeals to a law of expressive ‘economy’ to explain ‘conditional perfection’ from ‘if’ to something like ‘iff’.

8 Crediting Fogelin as a source of like standing for his explicanda of implicature, Horn’s HSs extend Fogelin’s subset, for which R equals |= by traditional definition, and they also include the set (believe, know) on which see Section 6, below. Fauconnier (1975), who seems to be the first to state that HSs in general invert under negation by simple contraposition, remarks that the ordering of HSs is actually ‘pragmatic’, in the sense that entailments can be relative to a context. This seemingly harmless contention, apparently distinct from Ducrot’s proposal, is casually endorsed by Gazdar (1979) and more fully by Horn (1989). It has given rise to analyses, e.g. Kay 1990, which—unlike Gazdar’s actual one—employ the word ‘entailment’ in a way that would make (lk) illustrate a designatum of it (cf. Merin 1997a, 1999a). If prepared to accept that all real gold is fool’s gold, given the right contextual assumptions, explore the open mind with silver (or indeed gold) and platinum.
mentative direction, as in

(2) Il n’en est pas satisfait, il en est enthousiaste.

Horn (1985) endorsed MLN, adding further motivation. MLN sustained HSs, since $Y \models X$ makes $\neg X \land Y$ a contradiction.

Examples: (a) Ducrot’s mode of reasoning might go like this. Suppose *He is happy with it* (A) is uttered as an argument for $H = ‘$We chose our present wisely’. Then *He isn’t happy with it* (B) is an argument against. Yet *He is enthusiastic about it* (C) is surely a stronger argument for $H$ than *He is happy with it* is,\(^9\) so a fortiori it is an argument for $H$. But then *He isn’t happy with it, he’s enthusiastic about it!* will present an argument against $H$ and one for $H$, and in a construction which does not admit but as a connective, but does admit argumentatively equidirectional *indeed*. And indeed, our reflective intuitions tell us that the first clause is not an argument against $H$ at all. (b) Horn’s mode of reasoning would be that $C$ entails $A$, which is incompatible with $B$ if $B = \neg A$, and so the asyndetic conjunction of $B$ and $C$ will be a contradiction. On either basic account, Horn’s or Ducrot’s, some interpretation of the negation morpheme (and perhaps of its prosodic environment) was wanted which did not have these respective undesirable consequences.

HSs supplemented by MLN would appear to explain puzzling facts of language.\(^10\) But their costs are too high and I have proposed an alternative\(^11\) which I should now like to propound at greater length and with arguments additional to those in the earlier statements. This alternative explicates and generalizes Ducrot’s ASs, but finds MLN entirely spurious.

2. Some Problems of HSs, Some Elements of DTS. The HS hypothesis, unlike its AS rival, leaves unexplained as a brute fact that Scales so often come in pairs. That Scales are simply given to us is plainly stated in Horn (1972/76) and Gazdar (1979). The AS doctrine, by contrast, can characterize negative Scales as those ordered sets of items which, upon substitution in simple and lexically compatible environments, will yield sentences having an intuitive argumentative direction inverse to that of sentences obtained by substitution of items from the positive scale.

\(^9\)We could write *happy about* instead of *happy with*, everywhere at a small loss of semantic parallelism. As things are, our Scale items will be *happy with* and *enthusiastic about*.

\(^10\)Cf. Horn op.cit; Kadmon (2001).

\(^11\)Merin (1994, 1999a,b).
When faced with a ‘foundational’ problem, people often react very sensibly with an ironic curl of the upper lip, which our quotation marks will surely have managed to represent. Since we must accept things as brutally given ultimately, the sensible person may ask, why not start right here? However, the problem, like any foundational problem which deserves the honour of disquotation, goes much deeper. That is, it goes right to the surface of description and imagination.

In the temperature examples given above, rising temperature determines the unmarked direction. The turning point for Scale membership of lexical items is the neutral or ambiguous item ‘tepid’ which, as Ducrot (1973) pointed out with sentences like *Cette soupe est tiède ou même froide* and *Cette bière est tiède ou même chaude*, can belong to both kinds of Scales. But what of composites such as *not hot* that might be lifted from *This motor isn't hot*?

The diagram in Horn (1989:244) (numbered 68) which resembles two stylized croissants nestling against one another, bespeaks the basic problem. One should wish to have *not hot* as part of the negative scale to which *cold* belongs. Horn’s hypothesis does imply just this: that *not hot* and *not warm* are entailed by each item in the negative Scale which is composed of lexicalized items. But this may not be enough to satisfy intuitions. Intuitively, and formally so for HSs, the extension of *not hot* also covers part of the intuitive temperature spectrum in the extension of positive items, notably of *warm*. So positive and negative Scales interpenetrate, deeply, not just in the tepid border area.\(^\text{12}\) This is what generates the feeling of unease or bewilderment which looking at the Horn croissants evokes in some readers. Another odd result of appeal to graphical heuristics, already encountered by Sapir, is a closeness of *warm* to *cool* which Sapir notes as being counterintuitive (cf. Horn 1989:245).

The AS doctrine of Ducrot, adumbrated in Ducrot (1972) and fully developed with an illustrative diagram in Ducrot (1973) has a clear intuitive membership criterion for negated items: *not hot*, whatever its countervailing *sous-entendus* (sc. implicatures) may do for its ultimate utterance interpretation, has argumentative direction towards low temperature or whatever low temperature would pragmatically mean in the context.

We can now see why we ought to feel less uneasy than we did before when faced with a diagram. Ducrot’s membership criterion for tempera-

\(^{12}\)“...the tail of each scale automatically extends through the excluded middle between the contraries into the rival scale.” (Horn 1989:243).
ture starts out with an intelligible physical scale, namely physical temperature, which plays a role somewhat analogous to the position coordinate axis in the phase space of classical mechanics. The set of argumentative potentials associated with intervals on this scale (assume the scale is bounded from above and below) plays a role akin to the impulse coordinate in phase space. The two do not confound one another, even if there is a non-arbitrary relation associating them. In the HS formalism there is no such dynamic or kinematic interpretation for regions, and hence one is left in some state of diagrammatic confusion.¹³

To advance the systematic argument, let me at this point move right forward to the positive proposal to be made. Doing so might render intelligible what is not perhaps completely obvious from Ducrot’s original approach and exposition.

Recall that in our temperature statement example we considered a conclusion whose aptness for adoption as a joint commitment was either monotone increasing or monotone decreasing in temperature.¹⁴ A decision-theoretic semantics [DTS] (Merin 1999a) will specify the pragmatic meaning relation in one of two closely related ways, reflected in the ambiguity or vagueness of the word ‘apt’. The first is descriptively conservative in restricting its formal ambit to doxastic terms. Here we should say, for example, that The motor isn’t hot will be a prima facie argument towards propositions at issue whose doxastic probability is monotone decreasing in temperature. Example: The motor will run all right for the next hundred miles. The second way takes account of Ducrot’s observation that the conclusion for which these kinds of arguments usually militate

¹³The virtues and drawbacks of Horn’s and Ducrot’s accounts are evenly distributed. Ducrot diagnosed, and may have been the first to do so, that negation reverses order of strength. But it was only Horn’s doctrine which had a formal account of this, as Fauconnier may have been first to realize.

¹⁴There are Scales of predicates which do not have an underlying physical scale—think of Fogelin’s pair of minimal Scales generated by goodness, badness and negation. But as long as we acknowledge good and bad as being ostensibly truth-apt predicates, we can relate them directly to conclusions. Someone’s aptness to being hired to prepare food in a cafeteria will, in an ideal world, decrease with their being found to be a bad cook, and increase with their being found to be a good cook. The claim (Horn 1989:242, apud Fauconnier) that Ducrot’s account of the order-reversing effects of negation in such cases will be ad hoc, whereas it will not be so when there is an underlying ‘objective’ scale is misguided. Ducrot’s account is either stipulative, and therefore ad hoc, for both cases, or for neither. The explanation to be developed applies to both cases.
in discourse is of an imperatival character.\textsuperscript{15} Example: \textit{Let’s keep going (on our motorized journey).}

Here a DTS will move from propositions to the notion of a \textit{random variable}, briefly, a function from states of affairs into a space of values. The values are usually confined to be numbers, though some definitions let them be any object, including \textit{acts or decisions}. Similarly, in decision theory, acts themselves are usually represented as random variables, i.e. functions from states to outcomes identified with numerical values (cf. e.g. Savage 1954).\textsuperscript{16} Suppose $\xi$ and $\zeta$ are random variables of the latter kind. $\xi$ might represent the act of keeping going for another 100 miles, $\zeta$ the act of stopping now. Let $E(\xi)_a$ and $E(\zeta)_a$ be their respective expected values for some agent $a$, perhaps a collective, $j$. The expectation of, say, $\zeta$ is the weighted sum of the values of $\zeta$ at each possible state of affairs (call it ‘possible world’), each value being weighted by multiplication with the probability assigned to the state of affairs. In general we do not know perfectly which world we are in. Assertions which we believe will reduce the set of worlds we think we might be in. So an asserted proposition $A$, if believed, rules out as wholly incredible all the worlds in which $\neg A$ holds.

When we condition on $A$, which is the Bayesian decision-theorist’s update rule for firmly believed information, we give $\neg A$ zero probability and distribute all its former ‘probability mass’ evenly among worlds in $A$, i.e. in proportion to their previous probability masses. But this may well change the expectations of our random variables and therefore, loosely speaking, the relative attractiveness of the acts they stand for. In our example, suppose $A = \text{The motor is hot}$ and let $E(\xi|A)_j$ and $E(\zeta|A)_j$ be the ostensible conditional expectations of acts $\xi$ (go on for 100m) and $\zeta$ (stop) for the ostensible commonwealth consisting of speaker and addressee(s). Then we shall no doubt find that ceteris paribus $E(\xi|A)_j < E(\xi)_j$ and $E(\zeta|A)_j > E(\zeta)_j$. By the laws of probability calculus, this finding implies $E(\xi|\neg A)_j > E(\xi)_j$ and $E(\zeta|\neg A)_j < E(\zeta)_j$.

Propositions, when they receive a probability assignment, are a special class of random variables called ‘indicator random variables’, i.e.

\textsuperscript{15}This is what motivated Ducrot to insist that the conclusion aimed for is not a proposition. There is a grain of truth in this, but it need not stand in the way of formal treatment.

\textsuperscript{16}But there is also a decision theory, due to Jeffrey (1965) in which acts, too, are propositions. Philosophers often adopt Jeffrey’s theory, because it retains both belief and desire literally as propositional attitudes. Economists tend to stick with Savage. I discuss the two approaches in Merin (2002a) in the context of language phenomena.
functions having value 1 when the proposition is true, and 0 when the proposition is false. The expectation \( E(\xi) \) of an indicator random variable \( \xi = \xi_B \) representing a proposition \( B \) is simply \( P(B) \), the probability of \( B \). Random variables in general are not propositions, and this makes them uncanny beings to most theorists of meaning. For present purposes we are well enough served by propositions and we shall explicate argumentative strength as evidential relevance, bearing in mind its relation to what I should call valuational or evaluative relevance. Evidential relevance will be a measure of the difference between conditional expectations of indicator random variables, i.e. a function of a difference between conditional probabilities. We will introduce these functions in Section 4 where they will be immediately applied in a way that warrants the investment in formal apparatus. Let us now return to the history of problems.

Horn (1972/76) took inspiration from Sapir (1944), a study of 'grading' expressions. Sapir investigates positioning at, before, and after a point of reference, motion towards and away from it, and immobility. He then explores the resulting nine-fold taxonomy. However—and consistently with a truncated description of Sapir's account of grading—the HS theory engages only the positional properties of grading expressions.

Not all Scales recognized as such by diagnostics come in pairs. The natural numbers \( N = 1, 2, 3, \ldots \) are the most glaring exception. There are no negative natural numbers, just as there are no sets with less than zero members. The most we might say here is that zero, the first ad-

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17 Disregarding probability, the logician’s name for such a function is the 'characteristic function' of a set, here a set of worlds at each of which we evaluate the proposition. A proposition here is understood as a set of such worlds.

18 Check on the back of an envelope, if this is momentarily unobvious.

19 For one, they combine like numbers, not like truth values. Semantics for fragments of natural languages in generalized random variables are given in Merin (2002a, 2002d). I do not presuppose this work in the present paper.

20 Horn (1989:245) refers in passing to Sapir’s upward and downward grading, reports his expression ‘notational kinesthesia’ and concedes the possibility of psychological superiority, but then focusses on graphic representations. Directional properties, I take it, relate to positional properties somewhat like first derivatives with respect to physical, causal time (c-time) relate to physical position. Ducrot, who does not refer to Sapir and between whose theory and Horn’s there are indeed close similarities, does engage an instance of Sapir’s directionality dimension. If we adopt the language of derivatives, the derivatives will be taken with respect to evidential time (e-time) which orders doxastic states or, more generally, states characterized by vectors of expected values which represent abstract asset positions or position changes of discourse participants.
dition to the naturals, is such a negative scale all of its own. Ducrot was wary about pronouncing anything definite about numbers. Horn was more forthright, but what he did say implies that natural language numerals are semantically meaningless.

Recall that the cardinal numbers 0, 1, 2, . . . form a partition (Frege 1884). The cardinal number 5, according to Frege, is the equivalence class of all classes that have five elements, exactly five elements. It neither includes another such equivalence class, say, one represented by 4, nor is it included in another. But to be ordered by entailment, HS items must form inclusion-chains or, synonymously, nests of denotata. Formally, and taking for granted that empirically diagnosed Scales always have a first element, we say that a Scale is a well-ordering under some relation R. Horn Scales are accordingly well-ordered by entailment or, equivalently, by set-inclusion. Thus, ‘X’ will basically mean ‘at least X’, so two means

(3) \( \{ \text{some } x \text{ in } \{ x \in S : x \geq 2 \} \} \),

where presumably \( S = \mathbb{N} \), with ‘exactly X’ obtained by implicature.

One problem with this account is that familiar reversals to ‘at most X’ as reported by Jespersen (1933) and exemplified by (1g) or by He lives on three if not two meals a day and those additionally reported by Horn (1972/76) (similar to 1h) remain unexplained. Likewise unexplained remain reversals which are found in permissive/concessive imperatives and in their modalized indicatival correlates

(4) a. Help yourself to \{some/two\}.

b. You may help yourself to \{some/two\}.

In the case of (4b) it is clear enough what the truth conditions are. The addressee will be in breach of the ostensible bye-laws of the social setup if he takes more than just some, say, most or all of the apples in the bowl.

\[ ^{21} \text{This does not mean, however, that we cannot get reversal phenomena. We shall meet some and a pragmatic account will explain them.} \]

\[ ^{22} \text{A well-ordering [w.o.] is a set ordered by a trichotomous, asymmetric, transitive relation } R, \text{ each of whose subsets has an } R\text{-least element. Trichotomy means that either } aRb, bRa \text{ or } a = b \text{ obtains. See any primer on set theory for more parsimonious characterizations of w.o.s. } \mathbb{N} \text{ is a w.o., the integers } \mathbb{Z} \text{ under their natural ordering or its converse are not, the negative integers are a w.o. under the converse ordering. I see no reason to think, as Caton (1966) and Hirschberg (1985) did, that there are Scales which are merely partially ordered, i.e. which include } R\text{-incomparable pairs of items. This little point is material if we recall that entailment is a partial order relation, whereas evidential relevance is total/trichotomic.} \]
More sharply, he will be in breach of the bye-laws if he takes more than two apples. By contrast, he will not be in breach of the law if he takes just a few or even no apples; and in the numerical case if he takes merely one apple or none at all.

Package deals (the lot or none) must be specially designated as being intended (e.g. by adding but no less in the (b)-case). In the (b)-case, whose effective act polarity is lexically fixed, even an abrasive exactly two does not force truth conditions which proscribe taking less than two. What it does, always, is to fix the non-negotiability of the upper bound, two (see Appendix 5). ‘Make no mistake’, is what it says.23

A naive objection to HSs will be that they predict the English expression at least to be semantically redundant. The reply might be that at least serves to cancel an ‘implicature’, to wit, the scalar implicature [sci] glossed ‘and no more’ or ‘but no more’. But the reply is costly, because no other expression of English seems to have only such a function. (But and also, which are employed to cancel implicatures, have other semantic content, too.) Furthermore, if at least simply cancelled the sci, it seems odd that it cannot be overridden by an explicit assertion of the latter’s gloss, for either variant of at least five {and/but} no more is unacceptable. We could yet wiggle out of this predicament by pleading incompatible speaker intent. But we should not if there is a more natural explanation.24

More troubling is the problem with numerals. Horn (1992) reports J. Sadock indeed objecting in 1984 that ‘at least X’ makes $2+2=3$ true, and J.D. Atlas replying in 1990 that numeral words are “lexically distinct” from “mathematical values” (i.e. ‘2’, ‘3’?) and are unspecified among ‘at least’, ‘at most’ and ‘exactly’. This reply, endorsed by Horn, is not remedial. Granted HSs,

(5) When you have five teddies and give Minnie two teddies, you have three teddies left

could be as false as

(6) Two and two is three

could be true. So suppose talk of quantity composition is no part of English. Then reversals under quantity comparison would still need added meanings

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23 For a detailed treatment of the permission problem, see Merin (1992).
24 See Section 4, below. See also Appendix 3 for a proposal which does leave at least its intuitable semantic content.
(7) ‘(some \( x \) in) \( \{ x \in S : x \leq 2 \} \)’.

Thus, numerals would be homophones (implausibly so) or underspecified. But, on current views, underspecification is logically disjunctive, i.e. set-union. Hence, in terms of truth-conditional denotation ([[\( n \)])], we should have \([[[n]] = (\text{some } x \text{ in}) \{ x \in \mathbb{N} : x \geq n \} \cup \{ x \in \mathbb{N} : x \leq n \} \) for any natural number \( n \) (perhaps with a redundant disjunct \( \{ x \in \mathbb{N} : x = n \} \) thrown into the bargain) and thus

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\text{(8) } \text{[one]} = \text{[two]} = \text{[three]} = \ldots = (\text{some } x \text{ in}) \mathbb{N}.
\]

3. Some Problems of ‘MLN’. MLN is described as rejecting an utterance as inappropriate in some way (cf. Ducrot, Horn, Kadmon, op.cit.). Specifically, the idea is that MLN is used to reject as inappropriate an utterance for any reason other than plain falsity. Ducrot, the apparent inventor of the notion, and who initially called it ‘polemic negation’, first hypothesized this device for presupposition rejection.\(^{25}\) But why should being false not be included among such reasons for rejection? If indeed not, this leaves a privative distinction, but one that, strangely enough, is not found lexicalized anywhere\(^{26}\) and is unexplained by scalar implicature. ‘MLN’ evokes Tarski’s ‘metalanguage’. But, in

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\text{(9) ‘Schwarz ist weiss’ is not } \{ \text{true/bisyllabic} \},
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‘not’ is plain, well-understood negation. The ontology of MLN, in contrast, remains unclear.\(^{27}\) By the standards of severely formal pragmasemantics (Gazdar op.cit.), not construed as MLN is without meaning.

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\(^{25}\)Certainly, the affirmative clause following the negation in (2), and similar clause in English, want exclamatory prosody. Feelings appear to run high, and Ducrot’s label ‘polemic’ may have reflected this quality. The exclamation mark wants explaining, nonetheless; and in Sections 4 and 5 we shall propose one explanation.

\(^{26}\)The closest we appear to get to it is when Horn (1989:427) endorses Maurice Gross’ claim that Fr. non (pas) immediately preceding its focus instantiates a distinct type of ‘contrastive’ negation, which Horn identifies with MLN. As Horn also shows, non pas trois may take continuations mais deux as well as mais quatre and we shall engage this fact below.

\(^{27}\)The point is not, of course, that MLN must play the very role which negation plays in Tarski’s metalanguage. The point is that the assimilation of labels will suggest that MLN is a notion with well-understood formal properties. But it is not. It is simply a joker in the pack of putative explanation, and current appeals to it are at their most insightful when they express, as occasionally they do, uncertainty about diagnosing it (cp. Kadmon 2001:76,150).
4. Act-Based Relevance Orderings [ABROs]. We assume that natural language meaning interpretation is predicated on paradigms of interpersonal commitment negotiation. They consist of elementary social acts (claim, concession, denial, retraction), their derivates and special cases e.g. assertion, admission, etc. (Merin 1994).

These concepts, reflected in the root modal Square of Opposition, imply allocations of warrant, temporal initiative, and preferences among agents.\textsuperscript{28} Actors claim what they prefer gaining; and what they concede is what they disprefer losing. Preference, when strongly enough specified, makes its objects indeed quantities (cf. Hölder 1901). Hence, with respect to preferences and a conventional orientation, a rational demand will always be ‘at least’, a rational concession ‘at most’. One should not rationally exclude getting more in line of preference, nor giving away less.

Let us begin with imperatives, where demands are apt to be concrete in prototypical examples. Thus, any claim for some $A$ in an ordered set $S$ of prospects, say, as in demanding

(10) Give me two marbles!

generates an ‘up-cone’

(11) $\geq A =_{df} \bigcup \{ X \in S : X \geq A \}$,

and any concession of $A$, e.g.

(12) OK, take two marbles!,

generates a ‘down-cone’,

(13) $\leq A =_{df} \bigcup \{ X \in S : X \leq A \}$

of prospects constraining possible compliance and uptake respectively.\textsuperscript{29}

\textsuperscript{28}Example: A claim by the speaker for a prospect $A$ (say, a joint commitment that the addressee give the speaker five dollars, with $A$ a proposition) will have preference for $A$ (as opposed to $\neg A$) assigned to the speaker who also thereby claims dominance, i.e. authority or other warrant or persuasive influence to sway the issue, and who assumes initiative. A concession to the addressee, by contrast assigns these three parameter values to the addressee. A denial, by contrast again, will assign the dominance parameter to the denier.

\textsuperscript{29}The relationship between down-cones and degree particles such as only and just is close, but in parts rather subtle. An expansive treatment, not found in the literature as yet, is wanted. Some discussion is found in Merin (1994), including an explanation of the curious speech act typing function of the German modal particle nur. (Ducrot 1973 already suggests that French seulement reverses scales.) When in need of a rough and ready periphrase for $\leq$, try imagining just with sentence scope (cf. also Appendix 2).
Lexical meanings $[A]$ which are cells of a quality-space or equipollence partition thereby acquire act-relative meanings which are cone-form sive ray-form constraints. Sets of unidirectional cones form inclusion chains, i.e. well-orderings under strict set-inclusion of sets of possible worlds or their more abstract correlate within a boolean algebra of constraints called propositions.

To make the transition to the realm of actions and transactions, note that a strategy, i.e. a mandatory contingency plan for various kinds of imperfectly foreseeable contingencies, is a constraint. What is of particular interest for present purposes is a joint or, more generally, jointly randomized strategy (Harsanyi 1977) which is to be adopted as mutually binding by speaker and addressee of a discourse. It is useful to assimilate even purely doxastic constraints, i.e. common convictions treated as common knowledge to this constraint category. An example from the world of finance would be a pair of asset positions reflecting the respective obligations which two persons have towards another. A non-vacuous transaction will be a change of asset positions. A change of common conviction will be another.30

Now, for one person’s claim to result in a transaction, there must be another’s concession. In the case above, abstracatly represented and illustrated by items (10) - (13), the net transaction prospect will be given as the intersection

$$(14) \; \geq A \cap \leq A = A$$

of dual, compatible ray-form constraints. This yields a rational construal of ‘upperbounding’ by scalar implicature.31

Adopting the industry standard terminology, though not its putative theoretical underpinnings, we can say that the meaning $\geq X$ is implicated

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30 The doxastic correlate of a complete strategy is a probability space, i.e. a probability function representing an agent’s doxastic attitudes to all the elements of an underlying algebra of propositions. The attitude to a single proposition $A$ may be seen as a very partial strategy. If $A$ is held for certain we can think of it as a pure strategy. If $A$ is held with a probability $0 < p < 1$ we can think of a randomized strategy, mixing a commitment to $A$ with probability $p$ and a commitment to $\neg A$ with probability $1 - p$. To see the point, let $A = \text{‘Combustion products cause adverse climate change’}$, imagine how $p$ conditions actions, expected asset positions, etc., and how people set about changing $p$, not excluding the case where it goes to 0 or 1.

31 I thus assume that such a notion is defined in imperative contexts, not just in assertoric contexts, as Grice’s doctrine might require if explicated strictly in terms of informativeness.
by unmarked, ‘claim’ type use of $X$, and the upperbounding scalar implicature is merely another, countervailing implicature.\footnote{Specifically, $\leq X$ will usually be negatively relevant to $\geq X$ when the proper up-cones and down-cones are non-empty.} The discussion of Section 2 should have made plain enough that we can treat imperative and indicational discourse in parallel for long stretches of theoretical accounting.\footnote{When it comes to operator scope, imperatives have much reduced variety. But this is readily explained and does not touch our argument here.} To simplify things assume that imperatives affect expected, extra-doxtastic values directly, while indicatives do so mediately through changing conditional expectations. The act structure of mediate and immediate acts, however, will remain more or less equal and I shall move freely between imperative and indicational examples.

This consideration brings us back to \textit{at least}. We say that \textit{five} ordinarily implicates the up-cone, while \textit{but no more}, which is grammatical in \textit{five but no more}, as usual serves to deny the net implicatum that quite possibly more than five is to be expected. That \textit{*at least five but no more} is unacceptable will then be explained as being due to outright contradiction among two assertions. That is, $[[\text{at least}]]$ maps its argument $[[\text{five}]]$ to a propositional abstract, i.e. to a type $<1,1>$ quantifier, $\geq \text{five}$, which is paraphraseable \textit{five, and possibly more}. The proposition duly formed, e.g. \textit{Five and possibly more cats walk}, is asserted or (using a better, more act-neutral term) stated.

Suppose now (as in Merin 1999) that a family of probability functions with representative $P^{j+1}$ represents the ostensible future joint doxastic commitments of speaker and addressee,\footnote{$P^{j+1}$ does this in assigning a degree of doxastic commitment to every proposition which an element of our basic algebra cutting up the world into mid-size packets. Recall that $P^{j+1}$ is a representative of a whole set of probability functions satisfying the (in)equality constraints we can actually elicit. So our imputable probabilities will usually be rather generous intervals.} and that a statement made in a context $j$ is intended to bring about such a future joint commitment state. Since $P^{j+1}(\text{morethanfivecatswalk}) > 0$ is part of the epistemic constraint to be imposed by our above example statement, while $P^{j+1}(\text{morethanfivecatswalk}) = 0$ is the constraint to be imposed by the assertion of \textit{but no more}, we explain the asterisk. Yet if $\geq [[\text{five}]]$ were the basic lexical meaning of \textit{five}, we should have no explanation. So, add \textit{‘at least’} to the list of problems for Hs which already contains numerals and, as will be verified in Section 5, negation.

Let us then consider denials and rejections. To deny a claim (as exces-
sive) [write ECLA for ‘excessive claim’] is to concede less than demanded, i.e. a prospect (possibly nil) in the boolean complement, within $\mathcal{S}$, of the up-cone, i.e. in

$$ (15) \quad ^{<}A =_d \bigcup \{ X \in \mathcal{S} : X < A \}. $$

Call $^{<}A$ the proper down-cone of $A$. Examples: Alpha demands of Beta Give me five dollars! Beta replies No. If Beta’s ‘No’ is adopted as a constraint on asset distribution, it will ensure that Alpha gets less than 5 dollars. Beta might consistently answer ‘Yes’ to a reduced demand for four dollars. On the other hand, the answer might continue to be ‘No’ for any non-zero amount of dollars. Similarly, now in the indicative mode, Alpha might claim The motor is hot. Beta’s No or sentential denial The motor isn’t hot will, if ratified, make it a joint doxastic commitment that the motor is less than hot. Conversational sequences analogous to those of the imperatival case are licensed.

Dually, to reject a concession $A$ as inadequate [write ICON for ‘inadequate concession’] is to claim (insist on) more than was conceded, i.e. to demand a prospect in the boolean complement within $\mathcal{S}$, of the down-cone, i.e. in

$$ (16) \quad ^{>}A =_d \bigcup \{ X \in \mathcal{S} : X > A \}. $$

Call $^{>}A$ the proper up-cone of $A$.\textsuperscript{35} Examples: Beta has replied Take four dollars in response to Alpha’s claim for (at least) five. Alpha, who won’t take ‘four’ for an answer, replies No! This leaves open that Alpha will take some amount greater than 4 for a satisfactory concession, conceivably the original five, but conceivably, too, say, 4 dollars and 50 cents. Similar things hold for the indicative case. Beta may have conceded The motor is warm (or: Three people attended) to Alpha’s claim that the motor is hot (or: that five people attended). Alpha’s rejoinder The motor isn’t warm, immediately followed by it’s hot! (or: Not three people attended, followed, say, by em at least four did! will reject Alpha’s inadequate concession.

The Jespersen (1933) observations on scalar negation,\textsuperscript{36} which are predicted by HSs (but not by Ducrot’s ASs) for the plain denial case,\textsuperscript{37} are

\textsuperscript{35}$^{>}A$ will be a proper subset of $^{=}A$. Geometers might treat the label as an alias for ‘unpointed up-cone of $A$', the point being $A$.

\textsuperscript{36}That not five is interpreted as ‘less than five’, though occasionally as ‘more than five’.

\textsuperscript{37}The case where not five is interpreted as ‘less than five’.
thus seen to follow. Rejection of a concession, a marked act in presuming a longer act-sequence, replaces recourse to MLN.\textsuperscript{38} We retain intelligible operands and operations, or rather operation. Our operation, even in the case of the rejection of an inadequate concession, is boolean negation and our operands are sets of possible worlds, i.e. propositions.\textsuperscript{39}

Some observations concerning putative MLN do find a ready explanation in these terms. Horn (1989:6.4) notes, as a first diagnostic for MLN, that MLN does not incorporate: *He is unhappy with it, he is ecstatic about it. But we know that prefixes \textit{un-}, \textit{im-}, \textit{in-}, \textit{dis-} in general designate polar opposites or indeed inverses of the Designatum of their tail, \(X\), and so will generally be elements of the proper down-cone \(<X\) correlated with, and disjoint from, whichever up-cone \(\geq X\) is generated by \(X\). The intuition that \textit{un-}, \textit{im-}, \textit{in-}, \textit{dis-} are apt for unprompted assertions (which would entail the proper down-cone generated by a denial of \(X\)) and do not lend themselves to anaphorically ellipsis coheres with the essentially, though sometimes virtually, anaphoric nature of rejection of an ICON. Horn’s second diagnostic is putative MLN’s inability to license negative polarity items (NPis) in its apparent scope \textit{He didn’t eat \{some/\textit{any}\} of the eggs—he ate all of them}. Again, this follows immediately from the above analysis, It does so for Horn’s English examples \textit{(some/any; sometimes/ever)} by simple appeal to standard truth conditions, and it will do so in terms of relevance structures in argument.\textsuperscript{40} We also go some way towards explaining the tendency for there to be intuited an exclamation mark on the positive clause following the rejection of an ICON. The folk label for repeating a claim, be it an imperatival demand or an evidential assertion, is ‘to insist’. This act type is part of an act sequence of minimum length 3, longer than for claims (length 1), and longer even than for concessions and denials (length 2).\textsuperscript{41}

\textsuperscript{38} As so often, some parts of this sequence may be virtual. However, no-one who adopts the Stalnaker (1974) theory of presupposition as entailment by a common ground context which is pretended to obtain prior to the utterance is in a good position to object to such a metaphysics of ostensibilia. See Gazdar (1979:106) for the anti-metaphysical position, and currently received accounts for its implicit rejection.

\textsuperscript{39} Kempson (1986) states that MLN is not needed for these examples, but offers no theory of the operands and is wrongly persuaded that MLN is needed for the ‘correction’ examples of Section 5, below.

\textsuperscript{40} Horn’s third putative diagnostic, association with obligatorily elliptic \textit{but}-clauses, will be addressed, briefly, in Section 5.

\textsuperscript{41} We find the same tendency for clauses rejecting a presupposition, but here the exclamative prosody attaches to the negation-bearing clause. The common factor,
Let us now turn from acts to the forces channelled by them. Recall that, when prospects are designata of imperatives, preferences on them can be represented by expectation-change-potentials for utility-valued random variables (rvs), i.e. as valuational relevance orderings.

For indicatives, a corresponding mood-specific measure of preference will be argumentative value. We explicate ASs as orderings by evidential relevance relative to a doxastic context \( <j, H> \). This is (i) a constraint on possible judgmental probability functions \( P^j \) that represent ostensible joint doxastic commitments and (ii) a proposition \( H \) ostensibly at issue (Merin 1999a). Evidential relevance

\[
(17) \quad r_H^j(E)
\]

of a proposition \( E \) to \( H \) in context \( j \) instantiates the special case of generalized relevance where the rv is of the indicator (0-1) type, and its expectation thus a probability.\(^{42}\) \( r_H^j(E) \) is positive if acceptance of \( E \) by conditioning \( P^j \) on \( E \) will increase degree of commitment to \( H \), i.e. iff

\[
(18) \quad P^j(H|E) > P^j(H).
\]

Comparative and negative relevance are analogously defined. E.g., where ‘warm’, ‘hot’,… are (increasingly) positive to \( H \), ‘cool’, ‘cold’,… will be (increasingly) negative to \( H \).

Ducrot’s ASs do not predict the Ducrot observations on reversal of ordering under negation. HSs and Fogelin’s subset do—as an instance of contraposition (\( \neg X \models \neg Y \) iff \( Y \models X \)). But ABROs predict likewise, though now on the basis of lexical meanings which are cells of a partition of an attribute space that induces a partition of propositions:

THEOREM: If \( X \) and \( Y \) are cells of a partition of the possibility space into propositions and \( r_H(Y) > r_H(X) > 0 \), then \( r_{-H}(<X>) > r_{-H}(\leq Y) > 0 \) (and \( r_{-H}(\geq X) > r_H(X) \)).

(Merin 1999a, Th. 7.)\(^{43}\)

\(^{42}\)See Merin (2001), where the labels ‘relevance\( u \)’ (\( u \) for ‘utility’) and ‘relevance\( e \)’ (\( e \) for ‘evidence’) are proposed for valuational and evidential relevance.

\(^{43}\)The Theorem with its easy proof (Appendix 1, below) was first presented at the Second Conference on Information-Theoretic Approaches to Language and Computation, London 1996, and in a talk ‘Negation and the Structure of the Lexicon’ at the ‘Sinn und Bedeutung’ Conference, Berlin 1997. Th. 7 in Merin 1999a states the first part. The second part (here stated in parentheses) is one of a family of results of which a variant is stated op.cit. for the case of relevance-orderings on sets of logical operators.
Recall now the intuitive problem of Scale membership for negated items. Neither ASs nor HSs explain in a satisfactory way why and how negation does change Scale membership. ASs do so intuitively, ABROs formally: negation reverses relevance polarity, always:

(19) \( \text{sgn}[r_H(\neg E)] = -\text{sgn}[r_H(E)]. \)

Thus, if it seems not to, polarity of its argument must have been reverse or nil.44

5. Resolving recalcitrance. Van der Sandt (1991) noted that boolean negation of the conjunction of another’s assertion, presuppositions, and implicatures leaves ‘not three: four’ coherent. But Horn (1985, 1989) diagnoses MLN and a typical intonation (Engl.: fall-rise) also in correction for pronunciation, connotation, etc. of items which are not deemed to be elements of HSs.45 Had van der Sandt tried to include these, he would have assumed arbitrarily large predictive powers.

We should, and can, be more specific. Note first how Ducrot’s 1973 MLN reflects a failure to recognize argumentation as part of political economy, whose repertoire of social acts includes not only inordinate claims, but also insufficient concessions, each to be rejected. He characterizes, perhaps even intends to define, MLN as “a rejection of an (implicit or explicit) prior affirmation/assertion (affirmation)” (Ducrot 1973:240). As Horn (1989:424) drily observes, one should not thus be able to characterize ordinary, ‘descriptive’ negation in the same terms, as Ducrot often does. But the failure of insight extends beyond cavalier labelling, into the phenomenology itself. Ducrot’s prefix gloss

(20) You don’t have the right to say that X,

for PN ‘not X, (but) Y’46 is apt not for his own example (2), i.e.

44This holds immediately for evidential relevance, i.e. for indicatival discourse. But something analogous holds also for utility or evaluative relevance, directly so when our prospect are propositions as in Jeffrey’s decision theory. If you prefer a possible item of preference, i.e. a non-degenerate proposition \( A \) prescribing how the world is to be, to the status quo, then you will disprefer its negation \( \neg A \) to the status quo (Merin 2002b).

45Appendix 2, below, looks at constraints on the intonation and interpretation of putative presupposition-rejecting instances of ‘MLN’.

46A lengthy act-theoretic analysis of the general distinction between occurrences of but and Fr. mais that translate, on the one hand, (as they would here) to Sp. sino and Ge. sondern and, on the other hand, to Sp. pero and Ge. aber is Ch. 3 of Merin (1996/99). See Horn (1989:402-413) for a summary of Ducrot’s and others’ work, associating MLN with the sino type, elliptic constructions.
(21) He isn’t happy (enough), he’s enthusiastic,
but for a non-instance
(22) He isn’t enthusiastic, but he is happy (enough).
It is (22) which sees the addressees exceeding their rights in claiming
enthusiasm, while (21) amounts to a reproach
(23) I’d be within my rights to say that 'Y' (Y = ‘he’s enthusiastic’).
Accordingly, (22) denies their excessive claim (ECLA), whereas (21) re-
jects their inadequate concession (ICON). Notice next that one is con-
ceding something in delivering in response to an exigency. Correction
examples, where apt for fall-rise, have the addressee (a pupil, foreigner,
...) failing to deliver what the speaker has rightfully and accommodably
demanded. Still, correction utterances can yet deny an ECLA. But then
the right gloss cannot be
(24) This just won’t do!/You can do better!.
It will be
(25) This is really asking too much!
—say, after the interlocutor has taken up an hour of our time with inade-
quate deliveries X of a proper name Y. And then X, in our last token
of not ... X, would no longer carry a fall-rise, but a rise-fall.49

6. Conclusion. ‘Horn scales’ properly extend the paradigm of ordered
sets of operators and predicates explored by Fogelin. This paradigm

47 The distinction ECLA/ICON is explicable not least by ostensible initiative dis-
tinctions. Ducrot’s affirmation turns out to mean ‘assertion’, as distinct from ‘admis-
sion’ and this is where the mistake lies in the present example.
49 The ability of French ‘contrastive’ non pas and presumably English ad-
determinative not to generate proper down-cones as well as proper up-cones (i.e.
licensing positive continuations with lesser as well as with greater Scale items) might
usefully be investigated in this context. As Appendix 2 shows, due control for prosodic
effects of real discourse anaphora is required. Our conclusion should, at any rate, be
that syntactically privileged occurrences of negation morphemes or morpheme strings
which exhibit this kind of bilateral openness are simply underspecified with respect
to act type, where underspecification may well come in degrees. There is no need to
obscure the fine structure of these phenomena with an opaque label.
hinged on Squares of Opposition and on their modest extensions by intermediates of subalternant sides. For example, \((\Diamond A, \Box A)\) extends to \((\Diamond A, A, \Box A)\), which validates the relations \(\Box A \models A \models \Diamond A\) in modal logic KT.\(^{50}\) For these paradigmata, an ordering by entailment was valid virtually by definition. Assimilation of the set \((\text{believe, know})\), also adopted by Gazdar, was still fairly unproblematic. Entailment holds here on construals which Fogelin, as a philosopher of language and human conduct, had second thoughts about, but which do have wide application and are standardly made in the informal metalanguage of epistemic logic. Whether or not to assimilate this pair of attitude terms to the entailment paradigm was, at any rate, a matter of discretion.

The real step beyond Fogelin involved predicate expressions such as numeral and temperature terms. This step was taken more or less simultaneously by two graduates in French linguistics, Oswald Ducrot and Laurence Horn. Temperature predicates and numerals yielded to the Horn/Ducrot diagnostics much as elements from Square of Opposition paradigms did. However, while Ducrot assumed that the diagnostics diagnosed relative argumentative strength in what was not necessarily conclusive argumentation, Horn assimilated the ordering relation to that which is also familiar from traditional Squares of Opposition, i.e. entailment. This choice of relation enabled him to predict important facts about the action of negation, facts which Jespersen (1933) and Ducrot (1973) between them had described, but not explained.

The price to be paid was high. The price was the assumption of a very odd hypothesis about the basic, semantic, lexical meanings of such predicates. Meanings now differed radically from what was to be expected from the constitution of graded attribute domains in the sciences. These are partitions, i.e., coverings into disjoint extension sets, give or take some fuzziness at the boundaries that can yet be taken care of by standard probabilistic devices.

This observation about domain structure is not confined to properly scientific usage. To intuit its appropriacy in everyday affairs, simply imagine how you would mark a thermometer if you had to use for labels no more and no less than the seven or so most common, non-numerical temperature predicates ranging from freezing over tepid to boiling. The analogous observation about numerals is even more striking. Here, indeed, the HS hypothesis implies a factual absurdity. HS basic meanings

\(^{50}\)See Chellas (1980). \(\Box A \models \Diamond A\) is valid already in logic KD.
for numerals differ from any intelligible concept of number.

To assume that ‘upperbounding’ by ‘scalar implicature’ is required to obtain the intuitive meanings is already somewhat implausible for temperature predicates such as ‘cool’ or ’warm’. But it is clearly untenable for numerals even for purely assertive use. Implicature, if it is to retain the motivation given in Grice (1989) and its developments (cf. Horn 1989), can operate only on utterance tokens of sentences that already have truth conditions. But, as Sadock already showed and rejection of a disingenuous response confirms, there would be no truth conditions to speak of for numerals on the HS construal.

HSs imply a theoretical contradiction in terms. They are intended to support a putatively conservative pragmatics, conservative in spite of its frequently attached claim to being radical. The pragmatics is to conserve intelligible logical meanings for basic connectives and the ‘logical’ quantifying determiners, meanings that remain stable under recursion. These meanings are rationalized by their fruitfulness in languages of inference which embody our gold standard of rationality, if any there is. But the extension of HSs beyond the ambit mapped out earlier by Fogelin and utilized by Gazdar saw the conservative determination of meaning turn into its contrary. Basic meanings in the extension were assumed to be weird, and indeed poorly or not at all suitable for languages of ideally rational discourse (cf. Merin 1999b). The pragmatics of implicature was to set them straight.

Even if the enterprise could have worked out in some world of myopic imagination, it should have been a lopsided affair. But it does not work out. The example of numerals—which are the canonical expressions for many of our most important generalized quantifiers of Lindström’s <1,1> type—shows that it does not, and conclusively so. The case of the modifying expression at least, which would become truth-conditionally pleonastic, is a further illustration of the problem. (In Appendix 3 we shall see that a partial reconstruction of HSs which retains a truth-conditional role for at least does so that the price of forgetting the existence of at most and less than.)

Moreover, the hypothesis of HSs entrained the adoption of the obscure notion of ‘metalinguistic negation’. Recourse to this construct was to overcome outright contradiction which ensued when Scale inversion phenomena had to be addressed. But already Ducrot’s original appeal to it as a solution to the analogous intuitive problem in ASs reveals a related confusion about the pico-political status of ASs. Argumentation
inherits the structure of more general social acts and is not, as Ducrot continued to think, an autonomous domain of rational conduct.

The hypothesis of ABROs allows us to preserve the economical insight that continued to identify the action of negation on the ordered sets called Scales as an instance of boolean contraposition. It reconciles it with two hypotheses.

The first of these is a conservative and intuitively appealing hypothesis on basic meanings for numerals, temperature terms and similar attribute domains.\textsuperscript{51} The second of these hypotheses is Ducrot’s intuitive proposal on the argumentative nature of the orderings.

The ABRO hypothesis is based on assumptions concerning elementary social relations which are ubiquitous in discourse about points at issue, points which are not moot, and which are addressed not only in the indicative but, as the case may be, also in the imperative mood. For the special case of indicative discourse the hypothesis relies on a rather old and, in scientific circles, standard explication of evidential relevance\textsuperscript{52}

\textsuperscript{51}Not a few writers have objected to HSs on this point, mostly on grounds of common sense (e.g. Carston 1988, who features a nice example: \textit{If there are three books by Chomsky in the shop, I'll buy them all}) or on grounds of poignant common sense. The latter description would fit Sadock (1984), whose trenchant objection I have reported after Horn 1992. (Horn’s report is, in fact, more pointed than Sadock’s own written version.) Atlas’s reported 1990 reply appears consistent with Carston and is referenced in Horn op.cit. as an ESSLLI summer school lecture. Granted that Horn’s report captures the relevant features of Atlas’s, none of these responses came with a theory that would explain, at the same level of intelligibility as Horn’s bare entailment thesis, at least those data which that thesis would explain if it could be sustained. Hence, they offered no alternative to it, even if Carston’s was apparently so intended. But to say that ‘context’ determines the appropriate reading, prior to implicature, on a base which has a ‘single sense’ is not saying much when the proposal for the sense of numerals $x$ specifies—in plain words that are either ambiguous, contradictory, or in need of trivializing assumptions—only what the sense is not to be: neither ‘at least $x$’, nor ‘at most $x$’, nor ‘exactly $x$’. To try and build a philosophical theory on such a slender base (give or take minor demurrals on putative points of technicality not identical with those just noted; see e.g. Recanati 1989, Barker 2003) is a mistake in method and in substance. The base for theory building would not be significantly firmer if we refined the original terms to this: to the bare and, as such, trivial claim that basic meanings are functions from contexts to the kind of meanings we want on empirical or theoretical grounds. (Functions in the usual sense are not elements of their own value range. So the claim does imply that there is now a difference, but this definitional assumption is all it implies.) Without further and formally committed specification of grounds, an appeal to context is no more informative, and probably less so about the appellant, than a more forthcoming appeal to the hand of providence.

\textsuperscript{52}Not to be confused with a homophone of more recent origin, referenced and
as doxastic context-change potential.

The taxonomic implication of the hypothesis of ABROs for the re-construction of meaning constitution is to shift the boundary between act-independent (relatively semantic) and act-dependent (and thence preference-dependent and hence relatively pragmatic) components of meaning constitution. What were putative basic lexical meanings under the ‘Horn scale’ construal are now already the outputs of non-trivially structured speech act or social act processes. It is the outputs of such acts, not basic lexical meanings, which constitute set-inclusion chains, i.e. ordered sets whose elements stand in relations of entailment (on the set-construal of entailment). It is these outputs of acts which are, in turn, the inputs to the interactive process of implicature that might be called conversational. In the present model, we make use of the same kind of act structure, namely by construing the Scalar implicature of an assertive claim as the anticipated, compatible admissive concession.

The act types are structured enough to ensure that these kinds of context-dependence are instruments of reductive prediction rather than a license for arbitrary forces to intervene in saving the phenomena. One way in which they are reductive is in allowing us to do away with occult notions such as ‘metalinguistic negation’. They also pay their way phenomenologically in allowing us to perceive what neither the hypothesis of Horn nor that of Ducrot or Fogelin had given cause to notice. This is a characteristic act structure of ‘correction’ examples whose English instances bear the fall-rise prosodic contour.

One may now ask why some ordered sets are more stable than others, when the ordering principle is to be relevance, be it evidential or valuational.\footnote{A question raised in discussion by Frans Plank and Christoph Schwarze of Konstanz University.} The answer is that many of the relations that make up our social and physical lifeworld are linear over a generous range of application. They are functional relations which preserve sums and proportions of inputs in their outputs.\footnote{This an instance of what is technically understood as monotonicity in the context of additive and multiplicative structures. It is these structures, not those of entailment lattices, which are structures of \textit{quantity}. If our word and concept orderings rightly carry the label ‘quantitative’, they do so on account of this additive and, as the case may be, also multiplicative constitution. Joseph Bayer of Konstanz University suggested on the basis of developmental data that in many cases we might be perfectly happy to have a weakening to qualitative stability laws, e.g. that addition increases...} The stability of such functional relations in

\footnotesize

\textit{exhaustively discussed e.g. in Merin (1999a).}
our daily life (e.g. those which factor through numerical quantity or temperature) makes for stability of relevance orderings. This is not least so, because our evidential relations are often at their most useful when they reflect causal relations. We all know, too, how surprised we are when reality springs some of its quaintier non-linearities on us. It is in such cases that typical, sedimentered relevance orderings lose their predictive powers for language and concept users. But where they retain them, the sciences of mind and conduct have a whole anthropology of relevances and anticipations to explore. They can now proceed in a manner less programmatic than that of a more impressionistic phenomenological attempt to elicit the constitution of the 'life world' (*Lebenswelt*). Very similar things can be said, and discovered, about preferred directions and about markedness hierarchies among elementary social act types.

In terms of Leibniz’s criterion for scientific world-making we are, I believe, better off with ABROs than with earlier proposals regarding the constitution of Scales and their expressions. We have reduced our reliance on arbitrary assumptions. We have expanded the plenitude of phenomena, i.e. of those bits of reality which make sense enough for us to even see them. It seems to me that success on the latter count is the chief criterion for the legitimate application of formal methods to objects of hermeneutic investigation. It is a criterion which ought to pull some weight even with those who reject the use of formal methods for an apparently good scientific reason. Their fear is that, in using these methods, one must ride roughshod over the small and really interesting details of human nature or culture. But the fear is based on a false premise: that proper use demands that we formalize everything we see. Doing so would indeed be a hopeless and altogether boring enterprise. The right premise, instead, is that we must be led by anything we formalize to see phenomena hitherto unseen. Our theory must have clear implications—it must be a theory—and it must be positively relevant to the size of our observation base.\footnote{Outlined by Alfred Schütz (1970) with much credit given to Husserl.}

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\footnote{Research towards the old and the new parts of the above argument, presented at the SFB 471 Kolloquium at Konstanz in May 2003, was supported by the German Research Council (DFG) under grants SFB 340 and SFB 471 and by the Thyssen Foundation. For helpful comments I am indebted to colloquium participants, to audiences at the venues mentioned earlier, to Hans Kamp of Stuttgart University, and size. This is entirely reasonable and realistic, and the science of psychophysics investigates appropriate transforms. But its reasonableness derives from embeddability in the ideal system which is reflects, for instance, in conservation laws.}
Appendix 1: Proof of the Theorem

We prove the Theorem in parts. First we prove Theorem 1, which embodies our main positive argument about scalar negation. It is demonstrated by means of a Lemma and a simple definitional fact about the relevance function. We might call Theorem 1 (or, for that matter, the Lemma) the Scalar Negation Theorem.

Next, we prove (Theorem 2) that the up-cone $\geq X$ generated by a positively relevant proposition $X$ is at least as relevant as $X$ is, and more so if there is at least one proposition $Y \neq X$ contained in $\geq X$, i.e. a $Y$ which is more positively relevant than $X$. Call this the Gross Claim Theorem. The Gross Claim Theorem provides a rationale for the phenomenon now commonly referred to as scalar implicature. The effect of scalar implicature, as explicated in Section 4 above, will be to reduce the Gross Claim proposition $\geq X$ to what might be called a Net Transaction Proposition which is less positive than the Gross Claim proposition along the latter's assumed direction of relevance. The possibility of such reduction follows immediately from Theorem 2.

To make the proofs reasonably self-contained, we now recall basic probability definitions. A function $P^j$, abbreviated $P$, is a probability function iff it is a real-valued function on a boolean algebra $\mathcal{B}$ (here: subsets of a set $\mathcal{M}$) such that for all $A, B \in \mathcal{B}$, $0 = P(\emptyset) = P(A \cap \neg A) \leq P(A) \leq P(A \cup \neg A) = P(\mathcal{M}) = 1$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ($\emptyset$ the empty set). Thus, $P(A \cup B) = P(A) + P(B)$ when $P(A \cap B) = 0$, in particular also when $A \cap B = \emptyset$. As usual, the probability of $B$ conditional on $A$ is given by $P(B|A) = \frac{P(A \cap B)}{P(A)}$ for $P(A) > 0$, else undefined.

Any probability function $P = P^j(\cdot)$ determines a unique relevance function $\text{rel}^j(\cdot)$ once we have settled on one of the many possible definitions of rel. The choice of definition will be a matter of convenience, provided our candidate functions are equivalent up to order-preserving transformations. This will mean that for any $P^j$, any pair rel, $\text{rel}'$ in the candidate set of relevance function types will satisfy $\text{rel}^j_H(E) > \text{rel}^j_H(D)$ iff $\text{rel}'_H(E) > \text{rel}'_H(D)$ for any $H, E, D$ in the domain of $P^j$.

Our proofs will be in terms of the log-likelihood-ratio (log Bayes factor) function for evidential relevance, notated $r$. This function has been employed e.g. by A.M. Turing and I.J. Good for signals intelligence analysis, and I have adopted it in Merin (1996/99, 1999a).\footnote{The reasons for adopting it are explained there and hinge on stronger than ordinal representational properties which are of no present concern.} Let $L_H(E) = \text{df} P(E|H)/P(E|\neg H)$ (known as the ‘Bayes factor’) and let $r_H(E) = \text{df} \log[L_H(E)]$, to some fixed base for the logarithm.\footnote{The logarithmic transform maps 1 to 0 and thus ensures, amongst other things,}
\(P(H)\), and \(r_H(E) < 0\) iff \(P(H|E) < P(H)\). We stipulate that \(r_H(E) = 0\) iff \(P(H \cap E) = P(H)P(E)\). This implies that \(r_H(E) = 0\) iff either \(P(H|E) = P(H)\) or \(\{P(H), P(E)\} \cap \{0, 1\} \neq \emptyset\). The second disjunct of the inclusively disjunctive ‘or’-clause takes care of cases where one or more of the conditional probabilities would become undefined when the conditioning event has zero probability. Thus, to all present qualitative intents and purposes, relevance relations represented by the highly intuitive function \(P(H|E) - P(H)\) will be faithfully represented by the function \(r_H(E)\).

In proofs we employ the following notational convention: For any \(Q \subseteq \mathcal{M}\), we represent \(P(Q|H)\) by \(q\), and \(P(Q\neg H)\) by \(q'\), where lowercase letters (e.g. \(q, q'\)) stand for real numbers. All instances of this abbreviating convention will be introduced explicitly or in the rewriting of an immediately preceding line. Recall also the set-theoretic characterization of negation by means of set-subtraction. If \(\mathcal{M}\) is our universal set, then \(\neg Q = \mathcal{M} \setminus Q\). One might think of \(\mathcal{M}\) as a set (or the set) of possible worlds, or in any other way that makes subsets of \(\mathcal{M}\) be or represent propositions.

**Lemma:** Let \(X, Y\) be cells of a partition \(\mathcal{F} = \{A_i : i = 1, \ldots, n\}\) of a set \(\mathcal{M}\) of possibilities into propositions linearly ordered by relevance to some proposition \(H \subseteq \mathcal{M}\). For all \(Q \in \mathcal{F}\), define \(\prec Q \overset{df}{=} \bigcup\{A_i \in \mathcal{F} : r_H(A_i) < r_H(Q)\}\) and \(\mathcal{M} \setminus Q \overset{df}{=} \mathcal{M} \setminus (\bigcup\{A_i \in \mathcal{F} : r_H(A_i) \geq r_H(Q)\})\). Then, if \(r_H(Y) > r_H(X) > 0\), then \(0 > r_H(\neg Y) > r_H(\neg X)\).

**Proof:**
1. Let \(S = \bigcup\{A_i \in \mathcal{F} : r_H(A_i) > r_H(Y)\}\) (union of propositions more positive than \(Y\))
2. \(R = \bigcup\{A_i \in \mathcal{F} : r_H(Y) > r_H(A_i) > r_H(X)\}\) (union of propositions more positive than \(X\) but less than \(Y\))
3. \(Z = S \cup Y \cup R = \bigcup\{A_i \in \mathcal{F} : r_H(A_i) > r_H(X)\}\)
4. \(s = \sum_i P(A_i|H) : r_H(A_i) > r_H(Y)\)
5. \(s' = \sum_i P(A_i|\neg H) : r_H(A_i) > r_H(Y)\)
6. \(r = \sum_i P(A_i|H) : r_H(Y) > r_H(A_i) > r_H(X)\)
7. \(r' = \sum_i P(A_i|\neg H) : r_H(Y) > r_H(A_i) > r_H(X)\)
8. \(z = \sum_i P(A_i|H) : r_H(A_i) > r_H(X)\)
9. \(z' = \sum_i P(A_i|\neg H) : r_H(A_i) > r_H(X)\)
10. \(L_H(\neg Y) = L_H(\neg [Y \cup S]) = L_H(\neg Y \cap \neg S) = P(\neg Y \cap \neg S|H)/P(\neg Y \cap \neg S|\neg H)\)

that irrelevance is literally zero relevance.

50 This is Kolmogorov’s well-known explication of stochastic independence, which is defined whenever \(P\) is, but which is less than fully intuitive. It yields the highly intuitive condition \(P(H|E) = P(H)\) when we can divide through by \(P(E)\).
\[
= [1 - P(Y|H) - P(S|H)]/[1 - P(Y\neg H) - P(S\neg H)]
\]
(note that \(Y \cap S = \emptyset\))
\[
= [1 - y - s]/[1 - y' - s']
\]
3. \(L_H(\neg X) = L_H(\neg [X \cup Z]) = L_H(\neg X \cap \neg Z)\)
\[
P(\neg X \cap \neg Z|H)/P(\neg X \cap \neg Z|\neg H)
\]
\[
= [1 - P(X|H) - P(Z|H)]/[1 - P(X|\neg H) - P(Z|\neg H)]
\]
(note that \(X \cap Z = \emptyset\))
\[
= [1 - x - z]/[1 - x' - z']
\]
\[
= [1 - x - (s + y + r)]/[1 - x' - (s' + y' + r')]
\]
(by 1. and union of disjoint sets)
\[
= [1 - y - s - x - r]/[1 - y' - s' - x' - r'].
\]
4. \(x > x'[r_H(X) > 0]\).
5. \(A_i \subset R\) implies \(r_H(A_i) > r_H(X) > 0\);
\[
hence P(A_i|H) > P(A_i|\neg H)
\]
hence \(\sum_i P(A_i|H) > \sum_i P(A_i|\neg H)\), i.e. 
\(r > r'\).
6. \([1 - y - s]/[1 - y' - s'] > [1 - y - s - x - r]/[1 - y' - s' - x' - r']\). \([2,3,4,5]\)
i.e. 
\(L_H(\neg Y) > L_H(\neg X)\) i.e. (taking logs)
\(r_H(\neg Y) > r_H(\neg X)\). (ok)
7. \(y > y'\) (since \(r_H(Y) > 0\)).
8. \(A_i \subset S\) implies \(r_H(A_i) > r_H(Y) > 0\);
\[
hence P(A_i|H) > P(A_i|\neg H)
\]
hence \(\sum_i P(A_i|H) > \sum_i P(A_i|\neg H)\), i.e. 
\(s > s'\).
9. \([1 - y - s]/[1 - y' - s'] < 1\) \([7,8]\) i.e. 
\(L_H(\neg Y) < 1\) i.e. (taking logs)
\(r_H(\neg Y) < 0\). \(\text{ (ok) }\) □.

Remark: The inequalities still hold if either \(R\) or \(S\) or both are empty. In this case, simply set the corresponding number pair(s) from \((r, r')\) and \((s, s')\) equal to the pair \((0, 0)\).

The Lemma immediately yields

**Theorem 1:** Let \(X, Y\) be cells of a partition of a set \(\mathcal{M}\) of possibilities into propositions which are linearly ordered by relevance to some proposition \(H \subset \mathcal{M}\). Then, if \(r_H(Y) > r_H(X) > 0\), then 
\(r_H(\neg X) > r_H(\neg Y) > 0\).

**Proof:** 1. The Lemma. 2. The definitional fact that for any \(H, E\), 
\(r_{\neg H}(E) = \log[P(E|\neg H)/P(E|\neg H)] = \log([P(E|H)/P(E|\neg H)]^{-1}) = 
-\log(P(E|H)/P(E|\neg H)] = -r_H(E)\). 3. For any real numbers \(a, b\), if \(a < b < 0\), then \(-a > -b > 0\). □
Examples: 1. If a statement ‘Kim has three credit cards’ is positively relevant to Kim is a person of substance, and the statement ‘Kim has four credit cards’ is yet more positively relevant, then ‘Kim doesn’t have four credit cards’ is negatively relevant to Kim is a person of substance, i.e. positively relevant to Kim is not a person of substance, and ‘Kim doesn’t have four credit cards’ is yet more positively relevant.

2. If ‘The liquid is warm’ is positive to Refrigeration is needed, and ‘The liquid is hot’ yet more so, then ‘The liquid is not hot’ is negative to Refrigeration is needed, i.e. positive to Refrigeration is not needed, and ‘The liquid is not warm’ yet more so.

The possible effect of scalar implicature subsequent to denial is not being considered in the assessment of relevance sign of the respective pairs of denials. But the relative relevance ranking within pairs, reversing that of the positive correlates, ought to persist even when scalar implicature impinges on denials. Conditions for scalar implicature to have an effect on relevance are demonstrated for the default case of a simple assertion by the Gross Claim Theorem:

**Theorem 2:** Let $X$ be a cell of a partition $\mathcal{F} = \{A_i : i = 1, \ldots, n\}$ of a set $\mathcal{M}$ of possibilities into propositions linearly ordered by relevance to some proposition $H \subseteq \mathcal{M}$. For $Q \in \mathcal{F}$, let $\geq Q = \bigcup \{A_i \in \mathcal{F} : r_H(A_i) \geq r_H(Q)\}$. Then (i) $r_H(\geq X) \geq r_H(X)$ always, and (ii) $r_H(\geq X) > r_H(X)$ if $\geq X \neq X$.

**Proof:** 1. Let

$W = \bigcup \{A_i \in \mathcal{F} : r_H(A_i) > r_H(X)\}$

$w_i = \sum_i P(A_i|H) : r_H(A_i) > r_H(X)$

$w'_i = \sum_i P(A_i|\neg H) : r_H(A_i) > r_H(X)$

$x = P(X|H)$

$x' = P(X|\neg H)$.

(For $w$, $w'$ recall that $\mathcal{F}$ is a partition.)

2. $L_H(\geq X) = L_H(W \cup X)$

$= P(W \cup X|H)/P(W \cup X|\neg H)$

$= [P(W|H) + P(X|H)]/[P(W|\neg H) + P(X|\neg H)]$

(recall $|W \cap X = \emptyset|$)

$= [w + x]/[w' + x']$

$= \sum_i w_i + x/\sum_i w'_i + x'$.

3. $\forall i : w_i:w'_i > x/x'$

(since $A_i \subset W$ implies $r_H(A_i) > r_H(X)$).

$\sum_i w_i/\sum_i w'_i \geq x/x'$

(even if $W = \emptyset$; for then $P(W|H) = P(W|\neg H) = 0$)

$[\sum_i w_i + x]/[\sum_i w'_i + x'] \geq x/x'$. 

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4. $L_H(\bar{X}) \geq L_H(X)$, hence, (taking logs)
$r_H(\bar{X}) \geq r_H(X)$. \ i.(ok)
5. If $W \neq \emptyset$, then [3] implies $r_H(\bar{X}) > r_H(X)$. \ ii.(ok) □

Appendix 2: ‘MLN’ and Inexistence

Would

(26) (η) The king of France isn’t bald—he doesn’t exist!

confirm the existence of ‘MLN’, as Horn (1989:424) maintains in critique of Lehrer & Lehrer (1982), who postulate an elided only or just after the leftward negation? It would not.

First, (η) will usually be sarcastic usage, and a rather heavy-handed instance too, one that will not mark out its utterer as person of supreme wit. Sarcasm instantiates a trope under which central communicative conventions, even truthfulness, may be broken more or less transparently. Unsarcastic or at least minimally sarcastic usage, as Ducrot (1972:38) confirms without apparently aiming to, would have at the very least cannot or can’t in place of not (see Merin 1999a). The modal offers a warning sign. Without it, the hearer will be led up an existential garden path. In either case, though, we have a straightforward fall intonation on bald.

Secondly, if one tries to minimize the existential garden-path quality of (η) that forces a sarcastic reading, and indeed make admissible a fall-rise on bald, one shall find it easier to do so with (η) than with

(27) (ηf) The king of France isn’t tall—he doesn’t exist!

The explanation will be that baldness is ostensibly a certain manque à être, positive to the kinds of conclusions to which utter inexistence is more positive yet. Being tall or indeed having any feature that will predestine for the human lead in motion pictures suitable for sustained family viewing is not. Having a fine head of hair (a property invoked in those very words by a most prominent owner-to-be of one, Bertrand Russell, 1918-19:[251]) won’t be positive to such a conclusion either.

By contrast, being hairy or hirsute, though each adding to the plenitude of physical being, should conceivably admit a fall-rise on their eponymous predicate expressions. This is because they might admit dispreferential commonplace evaluations, driven by their widespread countercultural or pongid
associations. A contemporary pretender to prime aristocratic status who instantiated these properties could already be somewhat immonde on account of doing so.

Existence, as the medieval theologians so acutely felt, is a perfection, perhaps the minimal perfection for any individual to have, but a perfection nonetheless.\textsuperscript{60} It follows that non-existence will be an imperfection, the ultimate imperfection we should say, and a natural candidate for membership in an ordered set of likeminded attributes.\textsuperscript{61}

By contrast, again, if we had didn't appear at Longchamps, or didn't show up, i.e. the negation of a property or action which would itself provide strong evidence of Being, a fall-rise on the corresponding predicate expressions would be impossible under the 'MLN' requirement of use in direct correction.\textsuperscript{62}

When the fall-rise is admissible, under circumstances where, as also required for 'MLN', the speaker is venturing to correct an immediately preceding, existence-implying assertion of royal baldness, the Lehrer paraphrase will be felicitous. To the extent that we develop the relationship between the concept denoted by 'at most', its dual, and items such as just or only, the doctrine of ABROs will explain the aptness of such a paraphrase. But it does also cover cases where a like principle operates without the lexical prerequi-

\textsuperscript{60} Anselm's famous characterization of the Divinity is aliquid, nihil maius cogitari possit (Anselm 1078). Here maius, whose first dictionary entry is 'greater', clearly means 'more perfect', being used interchangeably with melius 'better' in later sections of the Proslogion (Hick 1967). See also Russell (1905:[54]).

\textsuperscript{61} Some might feel that this foray into everyday poetics is itself but idle metaphor. It is neither. Existence in the most ecumenical, traditional sense is inextricably linked with the possibility of causal interaction. This is indeed what is implied by any realist and atomist reading of Kant's characterization, 'contained within the context of the whole of experience' (Kant 1787:B628f.). To exist is to be causally relevant. But to what? This is what is determined, in practice, by the interests and issues that shape our more parochial contexts. And on any useful theory of relevance, be it causal or evidential, there are degrees of such relevance. The rhetoric of ostensible relevance consists of attempts to impose representations of relevance on agents whose actions and reactions are driven by the usual range of motives worth a name.

\textsuperscript{62} In case of uncertainty about one's intuitions on these matters, or in case of outright disagreement, a long term interest in the phenomenon of negation in natural language will suggest the reasonable thing to do. This is to encourage a psychologist-phonologist to run statistically controlled experiments. We must, of course, ensure that we do not miss the semantic high harmonics and that there will be proper instruments to defer to. Experimental subjects should therefore be drawn from the population of people who have tested within the 99th percentile on the verbal aptitude section of the ETS Graduate Record Examination, say, from 1970 to 1990 inclusive. That population should make for reasonably good comparability with anecdotal intuitions of the average mature reader. Economical approximations to it might still be serviceable, though.
sites for insertive paraphrase being satisfied. These cases take us back to what I have called, for paradigmatic salience, the imperatival interpretation, whose relevance ordering is in terms of valuational rather than evidential relevance.63 Last, consider a properly non-existential variant

(28) (e) The king of France is not bald—(because) there is no king of France.

Here we do not ostensibly predicate inexistence in the actual relevant world (or, to speak more or less with Russell’s foil, Alexius Meinong, insubistence) of an entity somehow extant elsewhere. 64 Rather, we seem to deny that the property of being a king of France is instantiated. Try as we may, we find nothing that corresponds to the indefinite description, not even a ghostly substitute. With (e) the speaker appears to take Frege’s and, more specifically yet, Russell’s line on existence. This is not the line which admits a reading of ‘exists’ (perhaps the one which Meinong might have been referring to as existence without ‘subsistence’) that makes existence a predicate of individuals.65

Horn (1989:362) marks (e) for fall-rise on bald. But with this prosodic feature, the sentence is difficult if not impossible to pronounce for a competent speaker of English under the intended specification, namely of correcting the addressee. Competence-preserving pronunciation is possible only if the speaker

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63There is no assumption, of course, that indicatives do not admit of such valuational interpretations. Our factual discourse is not, for the most part, interested merely for argument’s sake, let alone disinterested. This is even more obvious for explicitly performative usage. A recognition of this fact need not conflict with the conservative and sound opinion that explicit performatives such as ‘We demand that you take immediate steps to transform public utilities into private utilities’ are in the first resort sentences that have truth-values, if any sentence does.

64In a nether world of fancy or forgettables. For the logically conservative way to differentiate (η) and (e), cf. Russell & Whitehead (1927:306f.).

65The latter idea has had something of a revival, and now very intricately argued too, in the ‘free logic’ developed and so called by Lambert (1963) and in the highly intensional work of Parsons (1980) and Zalta (1988). In particular, the idea extends to proper names or individual constants which do not, in a free logic, necessarily have to denote an existent in one’s domain of quantification. Observe then that nothing much changes for our examples if we replaced ‘the king of France’ with a proper name expression of suitable gravitas and style, say, ‘Louis Dissette’. (Cp. Russell 1905:54.) Whether or not the logical enterprise of differentiating forms of existence can be called successful as distinct from just coherent and, if so, whether the difference between the continuation clauses of (η) and (e) is to be captured in any of the logics latterly proposed, is something I have no answer to. What I do feel confident about is that the kind of data we consider bear on our pre-theoretical understanding of the notion of existence. This notion is apt, for better or worse, to sway the metaphysician or computer scientist to opt for one spiritual engineering policy rather than another when there are no overriding reasons of a different nature. Might as well know what tune you are swaying to.
is now explaining a prior utterance of his own, in response to an actual or
implied query, *Why not?*, fielded by that addressee who demands that a reason
be given. This prior utterance by the speaker will presumably have been a
dead-pan and existentially deceptive rendering of plain

(29) *The king of France isn’t bald.*

Its main stress will have been on *isn’t* and it would, in its turn, have been
made in corrective response to an assertion or presupposition of royal baldness
communicated by the addressee.

The fall-rise on *bald* in (ε) would thus indicate plain anaphoric uptake.
It may indeed legitimately do so when the first clause of (ε) is a perfectly
ordinary denial of baldness. Such denial would, for example, be consistent
with, and indicated by, an alternative continuation with a clause

(30) — *(because) his barber would have told the Daily Mail long ago.*

Sentence (η), by contrast, can bear a fall-rise on *bald* without such a proviso—
but readily so only in those uses which do admit of the Lehrer paraphrase.
The essential point of difference to (ε) appears to be that being non-existent
(or inexistent or insubsistent) is ostensibly presented as a predicate very much
like being bald, and is therefore suitable to being made part of an ordered
paradigm. There is indeed little rationale for such a paradigm to be an HS,
but plenty for it to be an AS or, as one should explicate and generalize, an
ABRO.

If the putatively presupposition-rejecting uses of all these sentences can
bear a reasonably characteristic intonation other than the deadpan default
prosody of sarcastic gardenpathing, it will be what is most widely known after
Liberman & Sag (1974) as the ‘Contradiction Contour’. The label, as Cutler
(1977) pointed out, is misleadingly narrow when read as a description. I should
prefer ‘Fishhook’ on account of the contour’s intuitable fundamental frequency
shape or ‘Insinuation Contour’ on account of its presupposition-presupposing
function. (See Merin & Bartels 1997, which also has phonological parsings
in the current industry standard and more references.) This contour, so the
phonological literature (Ladd 1980) warns, must not be confused with the fall-
rise, which remains phrasal in scope (cf. Bartels & Merin 1999). In Merin &
Bartels (1997) the Fishhook is analyzed as a sentence-scope fall-rise, distinct
from its phrasal sibling. This sentential contour is fully compatible with an
existence-preserving, flat denial of baldness, backed by visual, tactile, and
tensile evidence.

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Appendix 3: Logical Existentialism and its Discontents

HSs have acquired part-time predictive company for numerals in the form of an account advanced by Nirit Kadmon (1987, 2001).\(^6\) Point of departure is a reported verbal suggestion by Hans Kamp to analyze, say,

(31) Five cats are cool

within the DRT formalism (Kamp 1981). Kamp’s basic doctrinal assumption, as reported, was that *five* denotes the number 5, that it usually receives a reading that has the truth conditional import of ‘exactly five’ and that pragmatic factors, specifically an intimation of speaker’s incomplete knowledge of the world, will implicate or otherwise imply an ‘at least five’ reading.

Kamp’s analysis can be broken down into seven steps and spelt out in ecumenical idiom. (i) Declare a second-order variable \(X\), i.e. a variable ranging over sets, have it represented by a discourse referent, and have it existentially bound with scope extending over the whole formula in \(X\) to be constructed in the following steps.\(^7\) (ii) Demand that \(X\) be a set of cats. (iii) Demand also that \(X\) be a set of cool beings. (iv) Demand that \(X\) have 5 elements, i.e. cardinality 5. (v) Demand that any set \(Z\) of cool cats be a subset, proper or improper, of \(X\). (vi) Allow that, under usual conditions of use, the condition ‘any set \(Z\) of cool cats’ might be tacitly understood as epistemically relativized to mean ‘any set \(Z\) of cool cats (among the animals in the neighbourhood) the speaker knows of’.\(^8\) Recall from condition (i) that \(X\) is bound by an existential quantifier, \(\exists X\). The resulting existential assertion

(32) \(\exists X[\text{cats}(X) \land \text{coolbeings}(X)] \land \text{card}(X) = 5 \land \forall Z[\text{cats}(Z) \land \text{coolbeings}(Z)] \rightarrow Z \subseteq X\)\(^9\) (where \(Z\) is implicitly constrained to range over sets of entities known to the speaker)

represents the condition that a Discourse Representation Structure (DRS), as given by (i)-(v), is true in a model iff there exists at least one verifying

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\(^6\) I go into this account in some detail, prompted by, and owing thanks to, an anonymous conference referee who claimed this was an advance on the original HS doctrine.

\(^7\) The second-order analysis of plural entities anticipates a proposal by George Boolos to read second-order variables and constants as plurals. Kadmon writes \(\text{cats}(X)\) to indicate this fact; Kamp & Reyle (1993) have \(\text{cat}*(X)\), as distinct from first order \(\text{cat}(X)\), which also disambiguates \(\text{cool}*(X)\).

\(^8\) Kamp’s suggestion was made verbally. Kadmon’s report (2001:69) is that the discourse representation structure (i.e. the set of formulae that explicates the conditions just being given) should be evaluated with respect to a restricted domain of individuals relevant to the conversation, e.g. the animals in the neighbourhood. Rest assured that apparent pedantry on this minor point is not really pedantic.

\(^9\) With first-order predicates \(\text{cat}\) and \(\text{cool}\) this is equivalent to \(\exists X[X \subseteq \{x : \text{cat}(x)\} \land X \subseteq \{x : \text{cool}(x)\} \land \text{card}(X) = 5 \land \forall Z[Z \subseteq \{x : \text{cat}(x) \land \text{cool}(x)\} \rightarrow Z \subseteq X]\).
embedding of it into the model. Kamp’s formalization of (29) by conditions (i)-(v) is glossed

(33) Exactly five cats are cool

by Kadmon. Another way of paraphrasing Kamp’s proposal would be either of

(34) a. There is exactly one set of five cool cats.
b. There are exactly five cool cats.

Kamp’s construal is apparently defeasible. There are occasional readings of Five cats are cool of which the nearest paraphrases will be At least five cats are cool or, equivalently, Five, and possibly more, cats are cool. Such defeasibility was conceded by Kamp and was to be explained by invoking his epistemic modification (vi). This clause specifies, in the context of the others, that the five cool cats whose existence is asserted are all the cool cats which the speaker knows of. It thereby opens up the possibility that there might yet be cool cats which the speaker does not know about. But note now that adding condition (vi) changes the appropriate paraphrase to

(35) Exactly five of the cats I know of are cool.

There is now introduced an element of deictic definiteness which is absent from (i)-(v). Moreover, our particular paraphrase introduces a semantic asymmetry between the restrictor predicate cats and the ‘comment’ predicate cool. Such asymmetry will be absent from paraphrases There are exactly five cool cats among the {entities/neighborhood animals} I know of, but definiteness is ineliminable. Paraphrase (35) is shorter and it will reflect a verification protocol by which the speaker first identifies the set of cats and then subjects the elements of this set, i.e. (all) the cats he knows of, to the test for cool (see Appendix 4).

Barbara Partee, as reported by Kadmon, suggested that condition (v) be demoted to the status of a conversational implicature. Kamp’s reference in

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70 The universal condition on any Z is represented by an ‘Implicative Condition’ in Kadmon’s representation, but its import is that of a universally quantified sentence. Our representation in logicians’ lingua franca, close to the ‘Duplex Condition’ occasionally used in Kamp & Reyle (1993) makes this formal fact explicit.

71 Observe that the ‘exactly’ part is being induced by the universal condition, (v). The paraphrase is truth-conditionally felicitous in the spirit of Russell, but not compositionally: there is no specific designator for exactly appearing. It shares this property with the standard Frege/Russell first-order explication (‘There exist individual entities v, w, x, y, z, all pairwise distinct, each a cat and cool, and there is no other individual entity that is a cat and cool’). For the analogous case of two cool cats, the formula is $\exists x \exists y [\text{coolcat}(x) \land \text{coolcat}(y) \land x \neq y \land \neg \exists z [\text{coolcat}(z) \land x \neq z \neq y]]$.
clause (vi) to speaker's knowledge would be excised alongside and would, one presumes, become part of the conditions for the generation of scalar implicature (Gazdar 1979, Soames 1982). The effect of removing (v) as a constraint on the assertoric import of *Five cats are cool* is to re-open a possibility which it was designed to rule out. This is that there are verifying embeddings which are representable by another second order sentence

\[(36) \exists X'[cooleats(X') \land \text{card}(X') > 5].\]

Kadmon (and presumably Partee) point out that the amputated version of Kamp's proposal will account for the 'at least' readings of *Five cats are cool*.

Kadmon's empirical case against Kamp's description is conducted on the basis of an illustrative scenario (cf. Kadmon 2001:68) where either Partee's implicature (v) does not obtain or where Kamp's circumscriptive escape clause (vi) should need to be be invoked. Suppose I need to borrow four chairs for a meeting to be held in my office. You now tell me

\[(37) \text{Kim has four chairs.}\]

If you know that there are four chairs in Kim's possession, but are unsure of whether there might not be chairs of Kim's which you are unaware of, either clauses (i)-(iv) [Partee] or clauses (i)-(vi) [Kamp] will predict a reading according to which Kim has four and possibly more chairs. But now suppose you know that Kim has ten chairs. Then Kamp's variant (i)-(v), suggests Kadmon, will generate a false sentence, while Partee's will still be true. The assumption appears to be that an implicature to the effect of (v) will not be generated if the speaker knows that there are more chairs in Kim's possession.

At this point, I suggest, you ask yourself: Why did you, the phenomenological speaker who knows that Kim has ten chairs, say *Kim has four chairs*? Why did you not say *Kim has ten chairs*? The answer is obvious enough and indeed Kadmon has made explicit use of it in setting up the example. Four chairs, no less, but also no more, is what I need for my meeting. Obtaining fewer than four won't do; and we may surely fill in the example in line with life experience by assuming that getting any number of chairs greater than four will be a hassle or indeed a nuisance, and without compensating benefits. (Kadmon does not do this explicitly, but see what happens to your intuitions if you think otherwise.)

However, one page later in Kadmon's account, the implicit condition of need and its import are forgotten. Kadmon has determined that Kamp (who has no place for actual defeasibility) is in trouble and she generously offers him a way out. Sometimes, she concedes, (37) can be understood as

\[(38) \text{Kim has four chairs that she is willing to lend.}\]
So, if you happen to know that Kim has exactly four chairs that she is willing to lend, you will be truthful according to Kamp’s truth conditions.

But of course Kadmon’s generous offer comes with a sting. Suppose Kim is willing to lend not merely a particular 4-element subset \{a, b, c, d\} of her chairs, but any 4-element subset whatever. Then the uniqueness condition which Kamp’s scheme imposes\(^\text{72}\) will have Kamp predict the wrong meaning. It will predict that (37) will assert that there is a unique 4-element set of chairs that Kim is willing to lend. Yet the assumption was, presumably, that Kim is willing to lend any one of \(C(10, 4) = 210\) such sets of chairs, where \(C(10, 4)\) is the binomial coefficient. Exactly one set will be lent, but Kim is willing to let any one of the 210 sets be that set.

But even before we ponder the sting, we should realize that there is something wrong with the generous offer that preceded it. (38) is a red herring drawn across the tracks of short term memory. When I needed those four chairs, I presumably told you so; or else you read my mind. What I said or what you read will be tantamount to an utterance

(39) I need four chairs (besides those I already have, (for a meeting (etc.))).

This utterance of mine, made explicitly or imputed to me from circumstantial evidence, precedes your (37) and, if explicit, will have had main stress on four chairs.\(^\text{73}\)

Why can we be so sure of the latter? Because, in Kadmon’s suggested context, and to have the intended interpretability as being qualified ‘at least’, the word string four chairs must occur destressed in (37). (Check!) This is the standard phonological description for the gross phenomenological fact that main stress was on Kim. But destressing is a signal of anaphoricity. And we know from DRT and related semantics that discourse representation structures in such a framework are constructed from the contributions of all the speakers involved. We further know, when we analyze our data within the standard version of DRT, that my utterance or publicly inferrable thought (39)

\(^{72}\)Kadmon speaks of the ‘domain narrowing account’ and, in line with her paraphrase, does not mention uniqueness in connection with it. Given that uniqueness phenomena are central, titular parts of Kadmon (1987, 2001), the implicature that Kamp’s proposal does not impose a uniqueness condition (on sets!) is hard to avoid until this moment.

\(^{73}\)And if it was not made explicitly, a serious DRS will nonetheless have represented the relevance structure of my inferred desideratum. That such a relevance structure has been inferred is indicated by your stress pattern in the eminently vocal (37). And if one’s favourite theory of prosodic focus should not imply that such relevance structure is assigned (test the work related in Kadmon 2001:227-414 for such implication), then it is time to replace it with a better theory. For a positive proposal, see forthcoming work of mine, which is ready for vocal presentation in lecture format.
should have introduced a set-valued, existentially bound discourse variable \( Y \), constrained to be a set of four chairs needed by me.

My need is as intensional as the intensional verb *need* says it should be. But, intensionality of the requisite kind will be available even in our extensional framework. All we need to be sure of is that our world contains at least four chairs (in addition to those I already have). The big existential quantifier that binds \( Y \) is big and purely existential precisely because it does not imply that I myself am either able or indeed required to specify a particular set \{a,b,c,d\} of chairs that I need.\(^{74}\)

The rest of the story is obvious now. *Four chairs* in (37) is destressed because it is anaphoric, and if we take account of this fact in paraphrase, we shall paraphrase it as *the four chairs*. But which four chairs? By the rules of anaphora and of DRT, these are the four chairs that are variably designated by \( Y \) and satisfy the constraints on it. The variable \( X \) is to be unified with the variable \( Y \), whose existential quantifier’s scope extends dynamically. This is the quantifier that binds \( X \). Whichever four-element set \( C \) of chairs some particular embedding function \( \phi \) validating the existential Ramsey sentence\(^{75}\) of our discourse will associate with \( Y \), *the four chairs* will refer to that set \( C \).

The upshot is that our paraphrase for (37) in the original context provided by Kadmon (2001:68), must be

(40) Kim has the four chairs you need.

Unlike (38), this sentence will be true for any four-element subset of Kim’s chairs provided Kim has at least four chairs. Returning to Kamp’s proposal, whose import for the numeral determiner phrase is a definite description of a set\(^{76}\) we find we are done. The critique of his proposal rested on a misapprehension of discourse anaphora, whose treatment is the heart of the phenomenological part of DRT and related formalisms. We shall now see that the positive proposal of the (K)PK theory\(^{77}\) rests on a misapprehension of DRT’s more logical part.

\(^{74}\)To anyone who feels uneasy about intuiting second-order quantification, this fact will be obvious when we translate to the familiar first-order format of Russell and Frege.

\(^{75}\)Dynamic logical existentialism begins its life in Ramsey (1929) as second-order quantification.

\(^{76}\)As in

(41) Question: How many cool cats are there?

   Answer: There are the five cool cats I know of.

   Non-answer: “There are the five cool cats.

Clause (vi) paraphrases to the relative clause which licenses the definite article within the scope of ‘There are . . .’

\(^{77}\)My abbreviation for the modification, due to Partee and Kadmon, of Kamp’s
But first we must note where the (K)PK theory postulates a very general
distinction of numeral readings. Partee (1987) treats numerals are polysemic
and Kadmon agrees. Numerals are determiners in ‘referential’ *sive ‘argument’
positions that call for the existential construal. Here they behave as if pre-
fixed by a phonologically unrealized indefinite article. By contrast (Kadmon,
2001:69), they are adjectives when in ‘predicative’ position, as in

(42) The visitors were four lawyers and one doctor.

There will be no call for an existential construal, and numerals will thus retain
the meaning which Kadmon calls the ‘exactly’ reading. Kadmon observes that,
in the latter example type, the reading is indeed of the ‘exactly’ type. The rou-
tine truth-conditional construction of DRT supplies a basic truth-conditional,
assertoric meaning which is paraphrasable at least δ under (K)PK. Yet it
does not do so across the board, in particular not in predicative position. So
this appears to be a twofold advance on Horn. Stipulation across the board
would be widely replaced by explanation.

However, it would not be fully so replaced. The polysemic analysis is itself
stipulative. It turns predicative position numerals into adjectives and thus
exempts them from the existential quantification regime that is said to offer
scope for an ‘at least’ reading. Skeptics who think that type-shifting regimes
for polysemy are the latterday equivalents of the syntactic transformations of
the 1960s, may not wish to adopt it.\textsuperscript{78} Indeed, there are good aesthetic reasons
for not wanting to adopt it. The most important of these reasons is not that
the stipulation multiplies *ntia. (After all, who is to say a priori what is *pruet necessitatem?) Rather, it is the much more solid reason that the stipulative
solution leaves an interesting fact lying around unused. This fact is that the
subject position noun phrase in such examples is always definite, either as a
description or in virtue of introducing a finite set by ostension.

The following might be the procedure of analysis-by-synthesis which this
observation recommends to one who retains the Ramsey-Kamp account of
dynamic second-order quantification for numerals: (a) Let set-valued variables
be existentially bound ceteris paribus whenever they are introduced. (b) Adopt

\textsuperscript{78}Kadmon notes that the adjectival analysis of numerals goes back to Bartsch
(1973). Now Bartsch did indeed propose a category of ‘plural adjectives’ and put
numerals in them, illustrating with contexts such as the two houses. But the intima-
tion that this was not just creating a category whose only members were numerals
was buttressed by the claim that the red two houses was a well-formed word string
of English (1973:65). I doubt that red can pattern with first, second, next, middle,
and last, and so I doubt the credentials of the intimation. For an inconclusive but
sustained attempt to motivate an adjectival analysis, see Link (1998:103ff.).
Kamp's universal constraint and treat the 'visitor' kind of sentence as a second-order analogon to Russell's first-order sentence *The author of Waverly is the author of Ivanhoe*. (c) Introduce variables: X for the set of visitors, Y for a 4-element set of lawyers, and Z for a singleton set of one doctor. (d) Let W stand for the set-union of Y and Z. (e) Make X = W the main assertoric claim. (f) Remember that X is already definite in virtue of anaphoric or near-anaphoric accommodative pickup—for how else could that sentence be uttered? (g) Note that the uniqueness condition is represented by Kamp's Russell-style universal clause, which is inherited by W from X in virtue of the asserted equality (d). (Optionally, remember, for our past tense example, that the past tense of the sentence recurs to a past introduced earlier along with first mention of the visitors.) At this point, you are done.79 I am not myself endorsing this analysis, which relies on the existential quantification paradigm, as the best analysis to be given. The existential (K)PK analysis will in fact be seen to be unsustainable where it is purported to be explanatory. But as an argument against the polysemy claim it will do, whichever way it finally cuts.

Anaphora to a previous introduction of the first noun phrase referent is not obligatory for Kadmon's second kind of 'predicative' example, the sentence

(43) Bill, John, and Adam are two boys.

This is predicted to be false (as it must be), rather than true as an 'at least' analysis of two would predict. But of course, its coordinate subject NP specifies a set of three named individuals and is thereby already as definite as can be. An equative analysis along the lines just sketched might inherit the uniqueness constraint from it. One may, however, feel, as a matter of nagging reflective intuition, that two boys patterns like clever boys or overweight boys, and that it denotes the set of all sets which have two boys for elements. In this case the equative analysis of the copula should indeed give way to the set-elementhood analysis.80

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79 Kadmon (2001:74n) offers an example discourse *We have guests. The guests are three boys* and says that its 'exactly' reading is ensured by the uniqueness property of the definite NP. But a footnote is the wrong place for this observation, and the presupposition carried by the wording "will ensure an 'exactly reading here too' is as misleading as it is misguided, unless it is read, surely contra authorial intentions, as a casual or inadvertent admission that the account proposed in the main text is otiose. Placement and presupposition both suggest that, somehow, sentences like (42) or like Kadmon's original, *The visitors are four women and one man*, can be uttered felicitously without such an anaphoric context. But this phenomenological assumption is false for the language spoken by most fully competent users of English. If there is an empirical base to it, it will be a base of idiosyncratic intuitions. Reliance on them must severely circumscribe the descriptive import of the (K)PK theory.

80 The nagging intuition, and the absence of nagging in the first case can be oper-
Will this imply that *five* is polysemic after all? I see no reason to think so, even within the type-theoretic regime of Frege and Montague. Specifically, there is no reason to treat *two cats* as anything but a noun phrase (NP), wherever it occurs. If the type of NPs is \((et)t\),\(^{81}\) it can be so everywhere. After all, English has an obligatory copula which ought to do some work. Suppose, as linguistically applied Generalized Quantifier Theory traditionally assumes—wisely or not—that in *Two cats walk*, the sub-strings *cats* and *walk* each denote a set of individuals, i.e. are of type *et*. The NP has type \((et)t\) and *two* gets type \((et)((et)t)\). In *Two cats are cool*, the verb phrase string *are cool* should accordingly get type *et*. So *are* will denote a function of type \(\tau(et)\) where \(\tau\) is whatever the type of plain *cool* is determined to be. Suppose now, as Kadmon does and many scholars do, that *Felix and Felicia* denotes a set. Thus, it will have type *et*. Then, for *Felix and Felicia are two cats* to be a truth-valued sentence, i.e. of type *t*, letting *are* denote a function of type \((((et)t)((et)t))\) (or its modestly un-Curried equivalent \(((et)t)(et)t\)) will preserve NP status of *two cats*. With fancy typing for *cool*, we could preserve a uniform type for the copula. But why? There is really no better candidate in the English lexicon for type polymorphism than is the copula, or ‘glue’, as we might call it. Aristotle said of its Greek version that it was polymorphic, and current logic primers still distinguish equative from predicative readings of the copula. Being protean, as glue must be, would indeed give the copula some sort of *raison d’être*. Hence, if *two cats* denotes an NP everywhere, appeal to Partee’s putative type ambiguity\(^ {82}\) is no argument at all for differential treatment of NPs with regard to putative existential binding.\(^ {83}\) Below we show, as promised, that the existentialist treatment is the one that causes problems.

At this point, however, let me turn briefly from critique of the (K)PK proposal to the doctrine of ABRO’s, proposed in the main body of this essay and explored in Appendix 1. Note that the observer of ABRO structure will recognize that ordinary contexts of use for the predicative construction (42) are such that higher relevance for an expression-alternative involving, say, five lawyers and two doctors cannot be expected. But of course some explanation is

\(^{81}\)In Gallin’s ultimately perspicuous notation; Montague notated \(<< e,t,t, t >, t >\), computer scientists would notate \([E \rightarrow T] \rightarrow T\), where \([X \rightarrow Y]\) types a function from type X objects to type Y objects. Parsing accordingly.

\(^{82}\)Which Kadmon’s (1987, 2001) do not spell out.

\(^{83}\)And if one objects that *Felix and Felicia* is an NP, too, and yet has type *et*, one will still be under an obligation either to justify treating it as denoting a set (as Kamp and Kadmon do) or to justify treating *two cats* non-uniformly in its distinct occurrences. For mereological alternatives, which some scholars take to be notational variants of set-talk while others don’t, see Link (1998).
wanted why this particular sentence type does not readily lend itself to setting up a relevance (or other) Scale of numerical value.

The equational, inherited definiteness analysis is as good as any I can think of right now. For what the sentence does, on this analysis, is precisely what a typical subject-predicate construction does not do. It does not ascribe a general property, but prescribes identity.

When we change the setting and the quantities, things soften up, and, I believe, not just because of increased margins of perceptual error. Consider

(44) The listeners were two-thousand lawyers.

Assume this is said à propos a talk which has recently been given by a philosopher and who has, in virtue of this attendance record, finally attained objective relevance. The philosopher’s eulogist, who relates this crucial fact in uttering (44), can tag on if indeed they weren’t more numerous. However, no such tagging on is readily possible for the ‘visitors’ example even when it is truncated of the doctor and multiplied by any numerical factor that preserves the bounds of a report on a routine social call (e.g., as in The visitors were five lawyers). When numbers rise, we can re-insert our philosopher into the scenario, now receiving homage from the legal masses at home, and growing in importance with every extra visitor. And then the tag is admissible, too. (I say tag, because it’s not quite a suspender clause of the usual elliptic form. But it does suspend and cannot be replaced here by a tag with fewer in place of more.)

For the analysis of Feliz and Felicia are two (cool) cats an equative analysis is less plausible, as I have noted above. But here, of course, the very subject NP ensures that no reading which is not a contradictio in adiectio is admissible. So even in a context where the speaker scores the more highly in argument the more (cool) cats he can name, the ostensive exhibition of the subject NP set’s cardinality will ensure that no Gross Claim for that NP possibly denoting a set of more than two cats is sustainable. If, in a like situation, we offer The things around the corner are three cool cats we can again tag on if not indeed more of them. Here we end the excursion to ABROS and also the critical discussion of numerals in predicative position.

This discussion has turned on faults of the (K)PK doctrine that are comparatively minor. For even if we should allow that, somehow, the polysemy hypothesis for numerals was both tenable and relevant, a quick perusal of the range of examples treated by the (K)PK theory—or rather: attention to the complement of this range within the phenomenological universe spanned by Horn (1972/76) and the literature on generalized quantifiers (e.g. Barwise & Cooper 1981; Keenan & Westerståhl 1997)—should indicate right away that (K)PK would have at most one leg to stand on; which cannot provide enough support in the undulating fields of science.
Let us begin with the mildest of phenomenological challenges. Horn’s thesis saw numerals behaving in parallel with the positive temperature scale. The (K)PK theory says nothing about the parallelism (while the doctines of ASs and of ABROs do). Still, one might reply in defence of (K)PK that a unified treatment is more of a bonus than a basic demand to be satisfied. Ceteris paribus it will put a theory at an advantage, but it is a negotiable property.

Rather more deeply worrying, and already overstepping the code of scientific accounting practice, is the failure of (K)PK to address the thoroughly familiar data on scale inversion. These data are presented incipiently in Jespersen (1933) as well as in his larger works, and much more comprehensively so in Horn (1972/76, 1989, 1992:173) to whose original, non-patchwork theory they are an evident nuisance. They all concern phenomena where the intuitive reading for plain, unmodified five is paraphrasable ‘at most five’. There is a bland assertion somewhere in one or both of Kadmon (1987, 2001), that numerals and some do not have ‘at most’ readings.\footnote{A judgment which was also professed by the abovementioned conference referee.} If accepted, the first conjunct of the assertion would, of course, disqualify Horn’s intuitions to the contrary.\footnote{Horn (1992:173), concurring with Sadock (1984:143), denies the availability of analogous intuitive readings ‘at most some’ for the weak determiner some. Here I differ from Horn and Sadock. I do so outright in imperatival contexts underr the concessive/permissive reading, where ‘All right then, take some’ and its indicational, explicitly performative neighbour ‘You may take some’ rule out taking many and leave open taking none. In evidential contexts we can get acceptable Some people attended, if any with fall-rise prosody on some and to say that here we let the Scalar implicature do our main communicative work would simply be to stick an unenlightening label onto an interesting phenomenon. The relatively poor availability of such readings in evidential contexts will, however, also be plausible in view of the assumption of the AS doctrine, an assumption I adopt, that some has prima facie positive argumentative polarity. It is not obvious, as some may think, that the latter is parasitic on the logical explication of some as at least one or its plural variant at least two. I have defended the polarity doctrine in a talk entitled ‘The Measure of All Things’ presented at the Formal Grammar and Mathematics of Language’ meeting, at Helsinki University, August 2001.} What, then, is to be done? There are several conceivable responses. The agnostic response might be to classify the situation as one of conflicting testimony. One measuring instrument (Kadmon) delivers readings which differ from those of another pair of measuring instruments (Horn, myself). Another response might be to allow for different strokes for different folks. Kadmon’s observation would thus be correct for her dialect or idiolect, and Horn’s and mine would hold for our respective languages.

But neither response can get past the scholarly convention that detailed and well known prior data which are inconsistent with an assertion should at
least be mentioned and challenged rather than disregarded. Since Horn is a
source prominently cited and since his phenomenological judgment on these
matters was not so obviously flawed by artefact that it has been universally
discarded,\footnote{This is putting things mildly. I know of no challenge to his phenomenology for
that existential issue which actually engages any of his examples explicitly.} we can already be sure on procedural grounds that there is some-
thing seriously wrong at least with the presentation of the \((K)PK\) doctrine.
Still, a determined defence of \((K)PK\) might yet brazen things out under an
appeal to idiolectal pluralism.

However, this option surely is no longer available when we turn to the
uncontroversially semantic, lexicalized heartland within the family of deter-
miners. Here the judgements crucial to the argument are ultra-robust, they
have not been challenged, and there is also no way to try and demote the
phenomena to peripheral status.

Kadmon offers semantics for determiners \textit{five, at least five} and \textit{exactly five}. The constraint for the first is \(\text{card}(X) = 5\), that for the second is \(\text{card}(X) \geq 5\)
and that for the third is that for the first with Kamp’s universal constraint
added.\footnote{Kadmon’s truly elegant observation (cp. also Kamp & Reyle 1993:458) is that
condition (v) conjoined to the constraint \(\text{card}(X) \geq 5\) which replaces (iv) imposes
no upper bound whatever on the size of possible sets of cool cats in our world. But
note also that this observation, by itself, would speak as much for Kamp’s original
proposal as for Partee’s implicational variant. Indeed, it might speak more strongly for
it, because it will be consistent with an implicature (v) applying regardless of \textit{at least}
being there explicitly, and simply being outsmarted by \textit{at least}. The same would hold
for the analogous observation that will attend a treatment of \textit{More than five cats are
cool} by way of a constraint, \(\text{card}(X) > 5\). Moreover, if Kamp’s proposal is the correct
semantics for \textit{Exactly five cats are cool}, as Kadmon maintains, the proposal would
have to adduce extra-semantic reasons for explaining why \textit{Exactly at least five cats
are cool} is absolutely unacceptable, instead of being a moderately prolix way of saying
\textit{At least five cats are cool}. The conclusion ought surely to be that Kamp’s uniqueness
clause is not really a semantics for \textit{exactly}.} But there is no treatment offered for determiners \textit{at most five} and \textit{less than five or fewer than five}. And there seems to be a systematic reason for
the observed pattern of omissive discrimination against downward monotone
contexts.

For suppose we replace the constraint \(\text{card}(X) = 5\) by \(\text{card}(X) \leq 5\) to give
a semantics for

\begin{equation}
\text{(45) At most five cats are cool.}
\end{equation}

This will be compositionally analogous to the proposal for \textit{At least five cats are
cool}. That we should be able to proceed thus is what any compositional
treatment of generalized quantification should demand. No prosodic or other
syntactic constraint appears to rule out the move either. But now the (K)PK semantics tells us that

(46) *At most five cats are cool* is true if and only if there exists a set of five cool cats.

This is being excessively narrow-minded. However, and seemingly to make up, (K)PK predicts with excessive open-mindedness that

(47) *{Fewer/Less} than five cats are cool* is true if and only if there exists a set of cool cats that has fewer than five elements.

This implies that both of the italicised sentences will be true whenever any of *Five cats are cool, At least five cats are cool* and (extending the positives without harm) *More than five cats are cool* is true. It implies, for instance, that

(48) *Fewer than five cats are cool* is true whenever *More than five cats are cool* is true.

By ordinary lights this is a contradiction. And even if we suspect that the contradiction arises from a sophism, the (K)PK theory generates just what the victim of the sophism is supposed to find reasonable.

The failure of (K)PK is instructive not least for the light it throws on Horn’s (1972/76) doctrine, of which it may have been intended to be a re-explication (cf. Kadmon 2001:71). The doctrine says that *Five cats are cool* entails *Four cats are cool*. The reasonable rationale is that there will be four cool cats whenever there are five cool cats. But what this means, of course, is that, given any set $X$ of five cats, there will be five distinct subsets of four cats. So an existential analysis is hidden in Horn’s ‘at least’ lexicology if we are looking for a rationalization. But of course, we cannot interpret the above contradictions in an existentialist manner to remove their contradictoriness, e.g. by muttering ‘If there exists a set of more than five cats, there also exists a set of fewer than five cats’. The ‘also’ tells all. More sharply, these contradictions are derived from the truth conditions which the (K)PK doctrine would assign to each of the two sentences being related, for the truth conditions given to one of them are plainly false.

At this point, let me turn once more to the main body of the essay. It has been suggested,\(^\text{88}\) in response to examples (6) and (5) of Section 2, namely

(49) Two and two is three

and

\(^\text{88}\)By the conference referee mentioned before.
If you have five teddies and give Minnie two teddies you have three
teddies left,

that the (K)PK theory could readily handle such examples. The first sugges-
tion offered in support of this claim was that

Two teddies and two teddies put together gave us three teddies

be analysed as

\[ \exists X, Y[\text{card}(X) = 2 \land \text{card}(Y) = 2 \land (X \cap Y) = \emptyset \land \text{teddies}(X) \land \text{teddies}(Y) \land \text{card}(X \cup Y) = 3], \]

which is a false sentence, as indeed it should be. The second suggestion is that

I had five (teddies), gave away two of them, and was left with three

be analysed in line with

\[ \exists X, Y[\text{card}(X) = 5 \land \text{card}(Y) = 2 \land Y \subset X \land \text{card}(X \setminus Y) = 3], \]

which is true, and so the original sentence can be true, too.

But this response is not to the point and an experimentalist will see already
by inspection that it is not. The original examples of Section 2 do not carry
past tense morphology. They are in the atemporal present and intuitively
express generalizations. The proposed examples carry past tense morphology
and do not intuitively express generalizations. The morphological fact is indeed
significant. Let us consider the first example, which is the simpler of the two
and which already supplies an experimentum crucis. The nearest neighbour of
(49) which takes up the concrete, material-content idiom of (50) and of the
response example (51) will be the equally atemporal

Two teddies and two teddies are four teddies.

We might be tempted to ignore the reflective intuition that this is a general
statement rather than a particular report, which would rule out the (K)PK
analysis immediately. What might sway us to do so is that the sentence is true
in our world under the (K)PK construal because there are at least 4 teddies
in our world. (I have seen at least four of them side by side in a shop window
the other day.)

However, we should not be swayed. A slight variation (summing up the
Zeitgeist and, in doing so, proving itself to be a grammatical sentence with a
point) will be false under the (K)PK construal:

\[ \text{See already Link (1998:102) on variants Three men } \{\text{lifted/can lift}\} \text{ the piano} \]

which are noted as introducing existential and universal ‘force’, respectively.
(56) Two billion teddies and two billion teddies are four billion teddies.

There are not enough teddies, as yet, in our world, to verify this sentence under the (K)PK analysis. But it is surely true right here and now, as will be even larger, astronomically-sized variants.

Unless there is some reason to think that the suggestion was based on unfamiliarity with the (K)PK theory, and given that there is no (K)PK analysis of such elementary examples in Kadmon (1987, 2001), one must conclude that the HS theory receives no net support from it. Nor, indeed, does it receive such support from the (K)PK approach in general. To the extent that HS and (K)PK stand a priori in a relation of positive dependence, as Kadmon intimated and as our above discussion of the sophistical reasonings made plain, (K)PK will be a net liability for the HS doctrine.

I conclude that the (K)PK theory of numeral meanings, though it makes the sensible assumption that two means 2 at lexical root,\(^90\) does not explain what it purports to explain. Where it appears to predict exceedingly well, it predicts in the manner of Russell’s stopped clock which tells the correct time not just once, but twice a day.\(^91\) Nevertheless, the (K)PK analysis raises a number of interesting questions about the relationship between the meaning of numerals in linguistic or extralinguistic contexts of use and canonical verification regimes. It is these regimes and the attendant notions of burden of proof and burden of evidence which provide the link between truth-conditional analyses and the interest structures which take us from dynamic semantics into pragmatics proper. They are addressed in Appendix 4. But before giving logical existentialism another pass there, let us consider Kamp’s original suggestion in the same relatively superficial manner in which we have proceeded so far.

One way to block the mispredictions of (K)PK in downward contexts is to sew back on Kamp’s condition (v), i.e. to retransform (K)PK into plain (K).\(^92\)

\(^{90}\)Kadmon (1987, 2001) calls this the “exactly n” meaning of a numeral word n. This is harmless as long as it is understood not to be compositional for exactly n (see Appendix 5). I should prefer to speak of the natural or, non-judgmentally, of the cardinal meaning of n.

\(^{91}\)It may be felt that this is colourful imagery even at the best of times, i.e. when nothing has been overlooked. But I think imagery serves a mnemonic purpose which on occasions is in need of being be served. Surely someone must have noticed during the last fifteen years how defective the (K)PK theory is, and then have forgotten to write up the observation. Indeed, Link (1998:100-106) notes that numeral NPs usually have an existential reading, but that at most n has universal, and more to the point, negated existential ‘force’. Yet there is no connection made, nor, perhaps more importantly, a foundational rationale offered for the difference. Appendix 4, below, is intended to do so.

\(^{92}\)And though I have not heard or read any remark to such effect from Kamp, and
With the condition, we rule out the existence of sets of cool cats having more elements than $X$ has, which, in the crucial example types (43) and (45), is constrained to having, respectively, at most, or fewer than, five elements.

Condition (v)—to change our simile now to something more dynamic—is like the brake pedal on a vehicle propelled by a powerful engine. The vehicle is the application of DRT to the semantics of numerals, and the engine is Kamp’s verifying embedding construction, which is the heart of DRT.\footnote{In provisional retrospect, the historical achievement of this construction is the unification of Karttunen’s (1976) proposal for the introduction of discourse referents with Ramsey’s (1929) dynamically extending existential quantifier construction for discourses whose genre ranges from scientific theories to fairy tales.}

The (K)PK theory removes the brake pedal, indeed the brakes. There may well be reasons for wanting to do so. If one is a driver who wants to attain maximal speed fast, brakes are simply so much excess baggage to be accelerated. But equally, being a driver who enjoys the rush of acceleration to high velocity is not incompatible with having an intact instinct of survival. The way to reconcile the two desiderata, speed and longevity, is to confine the ride to a straight run, on level ground, with no oncoming traffic and with plenty of empty space for a parachute stop. This little allegory will fully explain the circumscribed phenomenological ambit of the (K)PK doctrine as it appears from the literature, and also the laconic judgment that ‘at most’ readings of numerals and some are unavailable.

Having noted that plain (K) performs where (K)PK misperforms, let us return to Kadmon’s (and in parts probably Partee’s) objections to Kamp’s original theory sketch. One of them has been dealt with already. It rested on setting up a context in which, allegedly, even Kamp’s domain restriction tactic will not work to license the intuitable ‘at least’. As we have seen, this objection—at least in its available form (Kadmon 2001:70f.)—relies for its plausibility on a faulty thought-experiment and on giving insufficient credit to the pragmatic nature of Kamp’s the escape clause (vi). ‘Known by the speaker’ wants replacement by ‘needed by the addressee’ in the crucial example.

The second objection is serious. It is the observation, familiar from Horn and in parts from Jespersen, that

(57) Kim doesn’t have four chairs

intuitively means that Kim has less than four chairs, if any. Under the usual boolean, sentence-scope semantics of negation, an ‘exactly’ construal will pre-
dict a meaning according to which Kim has either more than four or fewer than four chairs and perhaps none. Horn’s basic ‘at least’ proposal was, among other things, a response to this problem. If, as Kamp’s construction demands, negation rules out the existence of a verifying embedding for the DRS, it will indeed deliver the false prediction, because it will apply to the conjunction of conditions which share a variable X bound by the widest-scope existential quantifier.94

To attain descriptive adequacy, both proposals, (K) and (K)PK, will be forced, respectively, to dismantle and to re-instal the brake (v) on the existential engine on diverse occasions. The modification (tinkering?) will always be ad hoc to their descriptive purpose. A logician in the mold of Russell, to whom natural language was of peripheral interest only—that is: of mere curiosity value as a capricious device for hiding the true logical form of arguments—would have no problem with such a tactic. But then he would not worry too much about lay intuitions either. To the compositionally committed student of language, such a course of action is not open. This fact might well explain why Kamp did not apparently pursue the proposal himself.

I conclude that neither the original (K), nor the modified (K)PK existential theory of numerals solve the explanatory problem. Having concluded thus, it still remains to explain the relationship between the ABRO doctrine and the eminently robust and familiar intuitions which underlie constructions such as that of Kamp.

The basic idea here is to address the relationship between the demands made by a claim, which hearken back to speakers’ preferences, and the incentives required for getting the claim conceded. In the case of ideally epistemic or doxastic discourse, these incentives are evidence. Logic, particularly under its fundamental, primary proof-theoretic construal, is a formalism for specifying how compelling evidence for complex sentences is to be obtained by recursion on compelling evidence for their constituent sentences.

94It is not clear at all, thus, that Kamp’s analysis could be modified readily to allow what Kadmon does not even consider, namely application of negation to the output of Kamp’s pragmatic escape clause (vi). On the other hand, although I am presumably in disagreement with Kamp on the nature of the pragmatic principles to be employed and on the role of existential binding, such a layering of relatively semantic and relatively pragmatic principles is what I envisage in the ABRO analysis. We start with plain 5, go pragmatically to ‘at least 5’, and then apply negation to get ‘less than five’.
Appendix 4: Logical Existentialism, Evidentialism, and ABROs

To make this appendix reasonably self-contained, as all our appendices are, I will go over some material of Appendix 3 once more, though from a slightly different angle.

I begin the analysis of numeral meanings in discourse by taking a broadly epistemic approach to them. A proof-theoretic approach is a familiar special case of an epistemic approach.\(^{95}\)

Five, we acknowledge, is often understood to mean *at least five* when it occurs as part of a sentence. For someone to whom *five* means 5, the simplest explanation for this fact takes into account the speaker’s presumable epistemic position in cases where it is understood as tantamount to *at least five*. Suppose:

(58) (a) Speaker Alpha has established of five entities that they satisfy a given set of properties (say, being cats and being cool);
(b) there are known or suspected (however vaguely) to be more entities as yet unexamined by Alpha.

Then Alpha’s assertion, *five cats are cool*, should and will be understood with the tacit proviso that there might be more than five cool cats in the world or in some relevant subregion of it that Alpha has not exhaustively examined. This understanding is made explicit in the analyst’s paraphrase of Alpha’s claim or commitment according to which *at least five cats are cool*.\(^{96}\) Suppose next that the above conditions of verification are what Alpha is routinely held to have satisfied in backing of a claim of *five*.\(^{97}\) Then there will be no motivation whatever for a dual understanding of *five* as *at most five*. There are two reasons why not.

First, and most importantly, to make such a claim, Alpha would have to have examined the whole of the conceivably relevant space-time region. If such a region is not specified in advance, the region understood by default will be arbitrarily large. It might even be the whole world. Either way, we cannot rule out a region which is too large to have been examined by Alpha. So we cannot rely on Alpha having been able to verify that *no more than five cats are cool*.

Secondly, our conditions of verification have ensured that *at least five* has been verified. (This proposition having been verified does not imply that this

\(^{95}\)A very explicit epistemic treatment of proof in terms of knowledge states is Kripke (1965), the benchmark modal semantics for intuitionistic logic, which takes up model-theoretically Gödel’s suggestion to represent it as a classical modal logic with a primitive sentential operator “is provable”.

\(^{96}\)Usually we will assume that Alpha is himself aware of (b), and in this case intent will be imputable to him straightforwardly. But sometimes knowledge of (b) may simply be imputed routinely to Alpha by an act of interpretive charity when only the addressee is known to know (b).

\(^{97}\)This format will often abbreviate the corresponding full sentence.
is the whole meaning of Alpha's utterance.) So less than five is excluded, granted that Alpha can trust his methods of verification (and granted that we can explain away a sophism based on a claim attaching to HSs, which I will soon present). But this means that there is no chance whatever that the contextual meaning of five (consisting indifferently of assertion, implicature, presupposition, and whatever else there might be) in that context can differ from that of 5.

Since exactly five is equivalent to the conjunction of at least five and at most five, that reading will also be disfavoured. But it will not be quite as disfavoured. If there is reason to suspect that an exhaustive examination of the relevant domain has taken place after all, then the at most component (or an equivalent) will become tacitly operative, i.e. after the fashion of an 'implicature'. Indeed, once we admit such a possibility, it seems to me entirely reasonable that an addressee might entertain a probability distribution constraining the likelihood of such an exhaustive examination having taken place.

But why should we not have a fully dual story to tell as well, one that starts with five, moves on to an epistemic conjecture at most five and then, by way of an implicature at least five, ends up with something defeasibly co-intensive with exactly five? The answer lies in our insistence upon speaker Alpha's prior verification of a sentence. Intuitively put, Alpha makes a claim about the cardinality of a set of cool cats. In the case of empirical assertions such as that of our example, verification proceeds by exhibiting five instances of the complex predicate cool cat, which is presumed to be of fixed intension. Once that mission is accomplished, there is no possibility left that the set, nor indeed the worldwide universe of entities, might contain fewer than five cool cats.

Let us suppose—counterfactually—that this implicit procedure or condition of verification exhausts all our verificationist options. Have we then not arrived at the HS claim? It seems we have. The lexical meaning of five in all admissible contexts of use will be tantamount to at least 5. And it appears that there will be an entailment from five to four.\footnote{Which, however, has an analogue for Horn in the alleged entailment from hot to warm that is not motivated by a like procedure of verification.} Suppose there is. Then the sentence Four cats are cool should be true.\footnote{Note that we are talking about a relation of consequence among sentences, not among alleged paraphrases. It is such intuitions of consequence which are, alongside other intuitions of sentence or discourse acceptability, our basic data.} But equally, if we are serious about our generalized quantifier analysis for determiners and about intuitions of entailment we must hold that Five cats are cool entails No less than five cats are cool.\footnote{A synonym of less in this collocation is fewer.} Of course four cats are fewer (less) than five cats. So Less than
five cats are cool will also be true. (Under the translation: There exists a set
Y of cool cats which has less than five elements will be true.) But then, No
less than five cats are cool will be false. (Under the translation: There does not
exist a set Z of cool cats which has less than five elements, which is a false
sentence.) Yet if five cats are cool, then most certainly no less than five cats
are cool. So we have a contradiction.

This argument—conducted while studiously ignoring the official transla-
tions which I have suggested in parentheses—is a sophism. The sophistical
slight of hand took place when we translated Less than five cats are cool as
There exists a set Y of cool cats which has less than five elements. What we
should have done to preserve intuition is to translate it as Every set Y of cool
cats there is has less than five elements.

But the sophism exposes a very real problem. Under Kamp’s translation
procedure as modified by Partee and adopted by Kadmon, the locus of ap-
parent variation that reflects variations in the determiner is simply the car-
dinality constraint on the existentially bound set variable X. For plain five it is
card(X) = 5, for at least five it is card(X) ≥ 5. So, in the spirit of composition-
ality, we should expect it to be card(X) > 5 for more than five, card(X) ≤ 5
for at most five and card(X) < 5 for less than five. The first extension is fine,
the last two are not.

What then is it that tells us that the set Z in the intuitively intended,
non-sophistical, official translation of No less than five cats are cool, should be
subject to the condition that it is the set of all cool cats? It is this condition
which ensures that we render No less than five cats are cool as The unique
set Z of all entities that are cool cats has no less than five elements. It is this
condition, then, which ensures that the sophism will not go through.

Aiming to preserve compositionality we should thus require, as indeed we
intuitively do, that Less than five cats are cool be rendered The unique set Z
of all entities that are cool cats has less than five elements. These two transla-
tions would have to be the representations for assertions of the corresponding
sentences. Similarly No more than five cats are cool or At most five cats are
cool should be rendered as The unique set Z of all entities that are cool cats
has at most five elements.

Sophistical confutations pose few problems to the philosopher in the tra-
dition of Russell who is free to expose a ‘logical form’ hidden by appearances
that suggest the sophist’s proposed and intentionally misleading form. But
the analyst of natural language who believes in compositionality as a cordon
sanitaire against the forces of intellectual arbitrary must face up to these
problems. Algorithms for translation from syntactic word strings to inference-
warranting semantic representations often aim to proceed in such an orderly
manner.
Let us now take stock, in verificationist idiom, of what the existential analysis of *Five cats are cool* allows us to conclude. Almost all we can and must say so far is this: if Alpha is in position to have verified in the above manner that five cats are cool, Alpha will at some point have been in a position to verify that four cats are cool. All we can and must say in total is then obtained by adding or substituting the following: when Alpha is able to verify of a set $S$ of five cats that each of its elements is cool, Alpha is able to verify this of any non-empty proper subset of $S$. In other, more general words: if there is an entailment, it is from an ability to verify the larger to an ability to verify the lesser.

Now, once we are in the epistemic mode of talk about verification rather than in the purely semantic, Tarski-style mode of talk about satisfaction, we are talking about proofs. But a proof that 5 entities have a certain property is not, strictly speaking, also a proof that 4 entities, namely those in some (or in any) 4-element subset of our 5-membered set have that same property. If we look at a proof as in Hilbert/Gödel proof theory, namely as a syntactic object consisting of a sequence of sentences and capable of being represented by a Gödel number, then a proof of the proposition that 5 entities instantiate some property will be distinct from a proof that 4 entities instantiate that same property. What we can say instead, is that, under the conditions assumed, a proof that there exists a five-element set of cool cats establishes all the assumptions required to yield a proof that there exists a four-element set of cool cats. This consequence relation between proofs, that a proof of some $A$ can be ‘transformed’ into a proof of some $C$, is what constructivist a.k.a. intuitionist implication $A \rightarrow C$ is usually interpreted as having for its meaning. Kamp's (1981) method of verifying embeddings turns this proof-theoretic consideration into part of truth-conditional semantics.

But have we not, then, arrived at the HS doctrine after all? No, we have not, not even under Kamp's transition from proof-conditions to truth-conditions which carry no constructivity requirement. We have at best established that *There exists an $n$-element set of cool cats* entails *There exists an $n-k$-element set of cool cats*, and does not entail the negation of *There exists an $n+k'$-element set of cool cats* ($0 \leq k < n; 0 \leq k'$).

When we express the sentence (1) *Five cats are cool* as *There exists a 5-element set of cool cats*, which is the closest to a canonical English paraphrase that there is to Kamp’s representation composed of conditions (i)-(iv), we surely realize that we have not done full justice to (1). The reply here might be that the first sentence also carries condition (v) as an implicature, while its paraphrase does not. But, if so, what explains that the paraphrase does not carry this implicature? If the doctrine of conversational implicature *apud* Grice were true, this sentence should carry the same implicature in view of
the definitional criterion of non-detachability.\textsuperscript{101}

At this point one may perhaps begin to think that Kamp’s reported semantics (i)-(v) had a point after all. But this point came at a price. The semantics was recognizeably untenable because it faced the very problem about negation which had in part motivated the HS doctrine. (Perhaps this explains why Kamp never pursued his suggestion in writing). Apparently sound reasons counsel for Kamp’s reported assertoric component of exhaustiveness.\textsuperscript{102} Apparently sound reasons counsel against. Kant’s famous response to antinomy—for this is what we appear to be confronted by—was to diagnose a false presupposition. This must be our response, too. Part of, or an implication of this false presupposition is that we have finished our semantic homework on having considered verifying embeddings. But we have not.

So suppose, as a first step to proceeding further, that we admit the uniqueness or definiteness conditions diagnosed above for sentences \textit{Less than five cats are cool} and \textit{At most five cats are cool}. A warrant for doing so is readily found, but it raises a question that leads us ultimately into a new theoretical ballpark.

How, we ask, is an assertion of \textit{Less than five cats are cool} under its intuitively intended meaning to be backed by verifying evidence? If \textit{Five cats are cool} is interpreted as an existential sentence, and if \textit{Less than five cats are cool} is its denial, logic requires it to be a general, universal sentence. Our sophism rested on treating it as yet another particular, existential sentence. But this was wrong not only because it led to a silly result, but also because it was not founded on a proper analysis of verification requirements. The assertor of \{\textit{Less than}/\textit{No}/\textit{At most} \textit{five cats are cool}, each understood as good sense requires, will be expected to have examined the set of all cats or, at any rate, the set of all possibly pertinent cats and to have established of all but 4 (in the last variant: all but 5) of them that they are not cool.

It has been a commonplace at least since Aristotle that universal sentences, unlike existential sentences, are hard if not impossible to verify for non-trivially large domains. This predicament commends to our attention an argumentative and expressive option for a speaker who would wish to assert such a proposition but who is, in fact, unable to assert it. His inability manifests itself in an inability to deny outright a claim to the contrary which he finds dubious or at any rate worth doubting.

Suppose he is faced with an existential claim that \textit{Five cats are cool}, a

\begin{footnotesize}
\begin{itemize}
\item[\textsuperscript{101}] Not to be confused with non-cancellability. This reminder is added for the comfort and safety of the general philosophical reader.
\item[\textsuperscript{102}] It is not clear whether Kamp & Reyle (1993:305-482) retain this analysis. They appear to do so implicitly at p. 454f, where \textit{Fred likes exactly three Beethoven string quartets} is paraphrased “The set of [Bsq.] which [F.] likes consists of exactly three elements” but is then given a DRS representation which would also fit “There exists a set of [Bsq.] which [F.] likes and which consists of three elements”.
\end{itemize}
\end{footnotesize}
claim whose actual or expected backing he does not find compelling. Under
the rules of English discourse, he need not assent outright by tacit admission
or by uttering a vocal Yes. Instead, he can concede the claim provisionally by
uttering a proviso: If that many.

This expression is essentially anaphoric. To make it felicitous as a codicil
to an assertion in its own right, namely

(59) Five cats are cool, if that many,

we have to consider an aspect of the context which has not so far been ex-
amined. Why, we now ask, would anyone wish or think it worthwhile to utter
such a suspenseful sentence in the first place?

We said that the assertor, Alpha, of Five cats are cool will be deemed
to be under an obligation to exhibit on demand five cool cats. Let us now
assume that ‘to assert’ has its everyday sense, distinct both from ‘to admit’
and from the neutral ‘to state’. The distinction is particularly clear in a court of
law, where plaintiff, defendant, and disinterested witness are the default roles
respectively associated with these verbs. Under such conditions, the burden of
proof devolves standardly upon the assertor, i.e. Alpha. Suppose Beta admits
(assents to, concedes) Alpha’s assertion. Then Beta need not, in turn, exhibit
these verifying instances for as long as Beta does not himself assert the sentence
to a third party, say Gamma.

Beta’s assent may, however, be guarded. Under rules of evidence and de-
cision which allow a Scotch verdict “Not proven”, Beta may simply wish to
indicate that he has no evidence to the contrary. In this case he may respond
anaphorically: ‘If that many!’. He cannot however, or so it seems at first hear-
ing, respond ‘Five cats are cool, if that many!’. But we can prepare a second hearing by looking at the larger picture.
Suppose that Alpha, whose family name is Kool, and Beta, whose family name
is Unkool, are not simply conversing to entertain the linguist. Nor are they
disinterested witnesses playing a role akin to that of measuring instruments.
Instead, they are parties to an issue between them which has raised a point or
has been raised by such a point. Then we must ask: How does the cardinality
of the set of cool cats, which seems to be at issue, relate to this point?

It is not difficult to infer a point. Suppose the likelihood of adoption of a
certain course of action or conclusion is monotone increasing in the size of the
set of cool cats. The obvious point at issue (obvious once you have been made
privy to it) is Kool’s and Unkool’s joint ability to stay put and withstand
an imminent invasion of animated hot dogs. Suppose, therefore, that there is
given a family $F$ of numerical functions, $f_i$, monotone increasing in the size
$\text{card}(X)$ of the set $X$ of cool cats, and suppose there are given two thresholds,$\tau_1, \tau_2$ of values of the $f_i$ that trigger a non-linear decision function $\delta$. A value
$f(X) \geq \tau_1$ triggers $\delta$ into acceptance of ‘We stay put’ as a jointly binding

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commitment (a joint strategy) of Kool and Unkool, and a value \( f(X) \leq \tau_2 \) triggers ‘We run’. Kool and Unkool keep arguing until one of the thresholds is reached.

In this argument, there are two parameters to consider. I will start with the second one, in the epistemic order of things. This parameter is simply the cardinality of the set of cool cats (in the relevant neighbourhood). The first parameter, by contrast, determines or narrows down which subset of the set \( F \) of monotone increasing functions \( f_i \) is to relate the number or proportion of cool cats to the adoption or rejection of the proposition “We stay put”. This second parameter is apprehended in the intuition that \( five \) may be represented as being a relatively large number or else as being a relatively small number, each time depending on one’s interests. Someone bent on staying put will ipso facto consider or pretend to consider 5 a relatively large number of cats, relative that is to the purpose. By contrast, someone bent on not staying put may consider or pretend to consider 5 a relatively small number.

The two parameters are not independent in practice. The proponent of staying put might add a rider as in

(60) Five cats are cool, and perhaps even more. or as in
Five cats, if not indeed more, are cool.

The opponent might instead add a rider of his own as in

(61) Five cats are cool, if (indeed) that many or as in
Five cats, if that many, are cool.

Given our rhetorical situation, and probably even out of the blue, many people will have an intuition that the first speaker is presenting \( five \) as a substantial number (of things), while the second speaker is presenting it as an insubstantial number. What triggers the intuition out of the blue, if there is one? The answer is clear: it can only be the rider. And the reason why this should be so is obvious enough. The two factors, cardinality and its position on a scale of significance, are both positive to one outcome and both negative to the other.

Note now that it is the opponent, but not the proponent, of a strategy whose likelihood of being adopted is increasing in these factors who may add the rider \( if that many \). In adding the rider, the opponent, Unkool, is doing something which the proponent, Kool, cannot do, at least according to the rules of English. This is to cast doubt on Kool’s verification of the claim for \( five \). One may object that the rider can be added more smoothly if \( five \) has been replaced by \( only five \). But it \( can \), according to robustly replicable intuitions, be added without \( only \), and in the given situation. Unkool’s warrant for adding it will be clear, too. Unkool has searched the relevant set of cats (perhaps even the set of all cats, if superhumanly gifted) and has established of all but five
of them that they are uncool (i.e. not cool). Of these remaining cats, some or all may at present still enjoy the benefit of the doubt.

Once we have become aware of the way in which the burden of instantial proof can devolve upon the opponent, that is, upon someone who employs a property (cool) such that his objective function will be monotone decreasing in the size of cool entities, we see how the proposed PK semantics for numerals which yields an ‘at least’ reading turns out to be pragmatically conditioned after all. The rider if that many is a piece of denotational semantics on any standard account of if, that, and many. It is also a bona fide suspender clause, i.e. an instance of one of the principal diagnostics for HS and AS membership. Suppose, then, that we do take it as seriously as the seminal literature on presupposition and implicature has routinely done. This means that we agree that assertions cannot be suspended by such a clause, while presuppositions and implicatures can be so suspended.\footnote{The class of diagnostic suspenders is very much a proper subclass of the class of conditional antecedents. The blanket hedge if I am not mistaken is not a suspender, as can be seen from Only Muriel voted for Hubert, if I’m not mistaken. Here the hedge will hedge against the possibility that Tricia, too, voted for Hubert.} Then we must conclude that its admissibility in (59) implies that the proponent of We run, (i.e. the opponent of We stay put) who begins the argument with Five cats are cool is not asserting a proposition which implies that no less than five cats are cool. In current taxonomy, this latter constraint is at best an implicature or a presupposition.

To see how this articulates with the epistemology of verification, let us return for a moment to Kamp’s embedding paradigm. Suppose we took the epistemic position of Unkool, the would-be falsifier of Five cats are cool. Unkool starts with a set of cats, say the set $W$ of all cats who are on the neighbourhood voting register. This is a finite set of publicly known cardinality, $\text{card}(W) = w$. He submits any member of $W$ whom he can get hold of to the Kool-Kat Acid Test (KKAT). Any such cat who shows signs of dissolution under testing is determined not to be cool, or, as Unkool says, uncool. Specificity of the test for uncoolness is a 100 percent. No cat that is cool will be classed as uncool. However, the test is not a 100 percent sensitive. Some cats that are not really cool will remain resolute enough under testing to appear cool. Being a trained experimentalist and a counterexample to the proverb nomen est omen, Unkool is aware of not only of his means, but also of his ends. He is aware of the characteristics of his test, and he is attuned to the purpose which it is to serve. Both insights reflect in his choice of words.

Suppose $w =_{df} \text{card}(W) = 50$. Having had $w - 5$, i.e. 45 cats fail the KKAT test (now considered as a test for coolness), Unkool concludes: Five cats are cool, if that many. The suspender reflects, in parts, his puritan epistemology. The lexicology of the main clause reflects his sense of relevance. Even if Unkool
is being somewhat laconic—he disdains a subject-focus only—his choice of expression is well turned. Kool can do without a redundant syllable, and five cool cats is more to the point than forty-five uncool cats, for reasons quite independent of expression length. The issue does not turn primarily on the cardinality of the set of uncool cats, whom neither Kool nor Unkool consider constitutionally capable of becoming collaborateurs to the hot dog invasion forces. The uncool cats, they agree, are immaterial and the objective issue hinges entirely on how many cool cats there are who can constitute a prospective territorial guard.

In terms of the DRT formalism, we can say that there exists a verifying embedding for the sentence Forty-five (i.e. \( w - 5 \)) cats are uncool. We can indeed not rule out that there exists a verifying embedding for Forty-six (i.e. \( w - 4 \)) cats are uncool. But this does not change the fact that Unkool, for reasons outlined, has spoken of cool cats, not of uncool cats. And yet, given the conditions we have, there are certainly no verifying embeddings for Six cats are cool and there might not even be one for Five cats are cool.

We have thus established that we can have ‘at most’ type readings for numeral NPs even in the indicative mood, and even without constraints imposed by lexical co-text (Jespersen’s ‘live on’ example) or specialized social games (Horn’s golf example; on which see below for more), and in the very syntactic position which ought to enforce the dual reading, if any. Moreover, we have demonstrated how both types of reading depend on two types of verification procedures that reflect in distinct distributions of the burden of proof.

I conclude that even in the domain of indicatives, the phenomenological judgment that numerals cannot have an ‘at most’ reading (Kadmon 2001) rests on inadequate experimentation. Between them, Jespersen’s and Horn’s well-known examples cover triggering of ‘at most’ readings by lexical environment (‘live on’) and background knowledge (some constitutive rules of golf). The present example is a third kind and I see no way in which it could be dismissed as lexically or contextually idiosyncratic.

If Kadmon’s claim were to be applied in the domain of imperatives, the same stricture would apply even more obviously. Someone who is being permitted to take five eggs by means of an imperative (All right then, have your way: Help yourself to five) is not in breach of satisfaction conditions if he takes fewer than five. Here we have no truth conditions in any natural sense, and so our intuitions of satisfaction conditions are of paramount importance. And it is false again to argue, as Kadmon would, that a permissive Help yourself to some eggs, introduced as such by All right then, go ahead: will see the addressee in breach of obligation when taking none, but strictly speaking in compliance when taking all (hemmed in at most by the imperative analogue of a scalar implicature). But since the main debates of semantics and pragmatics are still
concerned with indicatives, I rest the case on the suspender example.

To assume that Kamp's embedding semantics for indefinites carries over to contexts in which the implicit verification procedure no longer applies is to make unreflecting use of a tool that cannot, by design, cut both ways. How sensitive it is to conditions of application becomes clear when we consider expression variants. If we rephrase to

(62) There exist five cool cats,

the mathematico-metaphysical main verb will suggest exhaustive consideration of the domain either by inspection or \textit{a priori} by complete induction, i.e. by an admissible form of deductive proof. Here we do get a reading straight out of Frege and Russell, namely one equivalent to

(63) The number of cool cats is five.

The everyday variant

(64) There are five cool cats

which, for true felicity, either wants a prepositional phrase (PP) appended or the import of such a PP deictically understood, say, a PP

(65) in the bar

still retains an epistemic exhaustivity condition. But now, unlike with the upper register expression \textit{exists}, the condition is restricted to the spatio-temporal location set indicated by the PP and the tensed copula.
Appendix 5: ‘Exactly’: Variance-Reduction of Random Variables

Compositional semanticists with a lexicalist credo may already have had doubts that condition (v) of Appendix 3 is a representation of the English word exactly. The irremediable badness of Exactly at least five cats are cool will have been distributional confirmation of this hunch.

To the best of my understanding, exactly will have the following abstract semantics. It denotes a function \( f \) which takes for argument a random variable \( X_P \) with a unimodal epistemic probability mass distribution\(^{104} \) \( P \) over possible actual values of a putatively objective variable whose mode (and perhaps, though not necessarily, also mean and/or median) is denoted by the syntactic rightward argument of exactly, say five. This function \( f \) maps the random variable \( X_P \) to a random variable \( X_{P'} \) which differs from \( X_P \) only in having an associated probability distribution \( P' \) which (ideally) concentrates all probability mass on the mode, or which, at any rate, when five is considered as denoting a discrete random variable whose probability mass function has its mode on 5, reduces the variance of this random variable, if it was not already zero, to (something close enough to) zero. (For terminology see any textbook on probability, e.g. Feller (1957).)

Empirical experience with estimation tasks will immediately tell us that, for integer numerical estimates based on simple counting or subitizing, the variance of the estimator random variable with probability mass function \( P \) will be zero under good visibility for sets of size 5 or lower. For higher numbers, the likelihood of counting error increases, the \( P \) distribution may acquire appreciable tails, though sometimes we may infer from procedure followed that there is only one tail, namely a trail of what might be found under stones left unturned inadvertently or simply so far. In such cases, exactly has an appreciable effect, as it will have in all size ranges when the numerical value is based on investigative procedures other than simple counting of elements.\(^{105} \)

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\(^{104}\)I ignore the continuous case of probability densities, which requires standard or non-standard treatments whose niceties are of no present concern.

\(^{105}\)There is the familiar phenomenon that large or largish numbers, at any rate numbers beyond the secure subitizing range, are often read exactly when they are not conventional approximators (based on orders of magnitude of our arithmetical or counting system) such as multiples of 5 or 10 or of a dozen for objective variable values up to, say, 100, and then multiples of \( 10^n \) \((n = 1, 2, \ldots) \) with \( n \) monotone increasing in the magnitude of the objective numerical variable. This phenomenon is, I believe, explained by the availability and non-use of these conventional estimators against the background of routinely expected measurement error. The core theorems of mathematical statistics give a priori estimates of the distribution of measurement variables given certain standard conditions. But there may be other factors involved in the phenomenon, too, such as comparative syllabic. Krifka (2003) offers a theory of the phenomenon based on competing desiderata (deemed given) and the domination

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Kamp’s condition, as far as its logical form is concerned, has few affinities with such a general semantics for exactly. This can be seen already from the fact that the latter semantics is perfectly evenhanded as regards its effects on overestimates and underestimates. The closest direct connection would be the epistemic condition which regulates application of the constraint, much as everyday uses of the definite article are regulated by it.

That exactly cannot have Kamp’s or, ultimately, Russell’s proposal for the logicist characterization of numbers for a semantics becomes more obvious yet when we observe that exactly does not simply modify determiner phrases but also prepositional phrases. Most interesting among these for demonstrating how far we are from Russell is the PP schema between $A$ and $B$ when applied to its best known model, plane geometry, and its topographical approximate models. To say that Ulm is between Stuttgart ($S$) and Munich ($M$) is to mean that Ulm is on the straight line segment that connects $S$ and $M$, excluding the end points, $S$ and $M$.

Suppose we have eliminated or ignored deviances from the straight path. Have we then exhausted the meaning of the sentence? In standard mathematical explications of betweenness, say Hilbert’s (which we have just given above) we have done so. But is not there a nagging feeling, readily put to the test if one were to ask people to put a flag $U$ for Ulm on a line connecting points $S$ and $M$ in a diagram, that $U$ is more or less halfway between $S$ and $M$? The feeling is converted into near-certainty when we consider Ulm is exactly between Stuttgart and Munich. Again, once we have dealt with possible deviations from the straight line segment, this sentence tells us that Ulm is half-way between $S$ and $M$. The vagueness about where exactly Ulm is located on the open line segment connecting $S$ and $M$, which prompts the non-probabilistic mathematical definition, can be represented by a family of random variables which have their mean or mode or median (or all three in cases such as the normal distribution) at half-way between. Exactly maps it to another random variable which concentrates all or nearly all its probability mass on the half-way value.

It is clear enough that exactly has no proper argument when the word modifies at least five. The condition \{x : x \geq 5\} (or an applicative derivate thereof) which it imposes never denotes, except in degenerate cases which rule out its use, a single value of an integer-valued random variable, and hence cannot denote the mode of that r.v., i.e. the integer number whose associated probability mass exceeds that of any other value of the r.v. The same goes for at most, less than and more than. So much for exactly, for now.

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as suboptimal of certain combinations of associated feature values, which leaves the observed patterns of distribution undominated. My guess is that the two proposals pick up complementary parts of the complete explanatory story.
Appendix 6: At last: ‘At least $X_{<s,t>}$’

The connection between \textit{at least} and the paradigm of elementary social acts outlined in Section 4 may yet appear accidental or at most incidental to a descriptively inclined student of language. Accordingly, I will delve more fully into the act-related properties of this morpheme string of English. In some respects the analysis is similar to one given in Merin (1994) for the German morpheme \textit{nur}.

This phoneme string does double duty. Most prominently, it is a focus particle of degree (translating ‘only’). However, distressed in non-focal position, it is a modal particle, one of a class of discourse-function expressions in which German is very rich and whose formal semantics, despite valiant efforts in the literature, is by and large a mystery. I suggested that modal particle \textit{nur} is sentence-scope and will usually indicate that the sentence is uttered as a permissive or invitational concession.

English \textit{at least} has a related distribution of functions. With sub-sentential focus, it occurs as a modifier of numeral and other portion determiners (\textit{some}, \textit{a few}). When taking sentence scope, it is separable by a prosodic or written comma when prefixed, or by commata when parenthetically infix. In either position it is paraphrasable\footnote{See Kadmon (1987:103a–32).} by \textit{At least it is the case that} $X$ where $X$ is an indicative sentence, i.e. a syntactic object which in Montague's intensional typology denotes a function of type $st$, i.e. a proposition. In this role, it is translated by German \textit{zumindest} and by French \textit{du moins}. By contrast, its occurrence as the modifier of numeral determiners (e.g. \textit{five}) is translated, respectively, \textit{mindestens} and \textit{au moins}\.\footnote{Interestingly, when the unmodified English determiner is the cardinal or (as I would say) portion determiner \textit{some}, at least German may translate indifferently as \textit{zumindest einige} and as \textit{mindestens einige}. However, only the former can undergo preposing of the main verb into verb-second position. Example: the tensed copular auxiliary \textit{sind} in \{Zumindest/\textit{Mindestens}\} \textit{sind einige Kalauer kahl}. The two translation options are accompanied by distinct reflective intuitions. Their interest arises in that they show a non-arbitrary relation between the act of claiming and the ‘at least’ reading of indicatives.} To establish the fundamental discourse function of \textit{at least}, consider the following dialogue between two non-descript individuals:

(66) a. Alpha: She hurt him.
   b. Beta: She didn’t kill him.

If they spoke truly, we know that a certain female hurt a certain male and that she did not kill him. Permuting the temporal order of (66a) and (66b) does very little to affect possible connotation, and leaves unaffected the portable information content. Next, we are apprised that Alpha and Beta are desk
operatives of the Cold Comfort Bureau, a non-profit-making, tax-exempt organization and equal opportunity employer. We still do not know precisely whom they are talking about, but given what we know about Alpha and Beta, we can be sure enough of this: (i) The female in question is a hitwoman in the employ either of the bureau or of a competing organization, known to Alpha and Beta as a No Good Org, that enjoys charitable status elsewhere and is an equal opportunity employer, too. (ii) The male in question is another employee either of the Bureau or of the competing NGO. (iii) Alpha and Beta are assessing an encounter of the two employees in a recent theatre of operations.

But which of these individuals is in the employ of which of the two employers? Given the unambiguous job description of the female protagonist, and being morally certain that Alpha and Beta subscribe to traditional family values, we can learn all we need to know from each of the following variant dialogues:

(67) a. Alpha: She hurt him.
   b. Beta: At least she didn’t kill him.

(68) b. Beta: She didn’t kill him.
   a. Alpha: At least she hurt him.

Dialogue (67a/67b) tells us that he is the CCB employee, and she from the NGO. It also tells us that the encounter, in Alpha’s opinion, went badly, that Beta concedes it did not go well, but does maintain that it did not go as badly as it might have done, or equivalently, that it went better than Alpha might be suggesting it did. (We might now suspect that Beta had been in charge of CCB field operations at the relevant time and place; or else that we are witnessing a verbal exchange of manifest remorse for manifest consolation, but in this case with Alpha having been in charge.)

Dialogue (68b/68a) tells us something quite different. It tells us that she is the CCB employee, and that he is from the competition. It also tells us that the encounter, in Beta’s opinion, went badly; and that Alpha concedes it did not go well, but also maintains that it did not go as badly as it might have done, or equivalently, that it went better than Beta might be suggesting it did. (We might now suspect that Alpha, ... etc. mutatis mutandis.)

What else can we conclude as far as the meaning of sentence-scope at least is concerned? The essential point for present theoretical purposes is this: A truthful, undissimulating utterance of At least S, where S is a sentence in the indicative mood, indicates that S is positively relevant to that element of a binary issue (H, ¬H) which the speaker ostensibly prefers to the other element for adoption as a joint epistemic or deontic commitment of speaker and ad-
dressee. In other words, $S$ is a essentially a claim, rather than a concession.\textsuperscript{108} Of equal importance, though less immediately, is a further condition. $S$ will be a less than maximally strong claim in terms of the \textit{a priori} options of possible claims available. In example (67b), the strongest claim Beta should have wished to have been able to make (granted that the CCB employee was not himself a hitman) was that the hitwoman from the competing NGO did not hurt the CCB's own man, which in this case surely entails that she did not kill him. In example (68a), the strongest claim Alpha should have wished to have been able to make (granted that the CCB employee was a hitwoman) was that \textit{she} killed \textit{him}, which in this case surely entails that she also hurt him.

In each example, the utterer of \textit{At least} $S$ appears resigned to the fact that some preferred $S'$ cannot be claimed and that indeed its negation has been verified. Granted this, \textit{at least} seems to have lost its distinctive denotational feature of indicating that the up-cone constituted by the present gross claim is a disjunction of its base element (here: $S$) and some non-empty set of strictly preferred eventualities $S'$ (etc.), i.e. of a non-empty proper up-cone. This is no surprise, for the claim made with $S$ is a counterclaim—as it were anaphoric to an assertion of a negative whose sentential form is indistinguishable from that of a denial of a prior claim.\textsuperscript{109} The qualifier \textit{at least} does, therefore, play a denotational role, too, namely one which is analogous to that played by an explicit qualifier \textit{at most} when it heads someone's concession of a claim which either implicitly or by explicit addition of \textit{at least} is for an up-cone generated by the base item designated in the claimant's utterance. (67a) leaves open, save when we grant a scalar implicature, the possibility that the NGO hitwoman did kill the CCB operative. Similarly (68b) leaves open the possibility that the CCB hitwoman didn't even hurt the NGO operative. In each case, the reply rules out this possibility.

However, even in its role of indicating a limitative counter-claim, adsentential \textit{at least} retains the function of indicating that there are or were outcomes, counterfactual outcomes, to be sure, which are strictly preferred by the speaker. In other words, the expression \textit{does} indicate that the sentence in its scope denotes a proposition which is an element of an ordered set of propositions and which generates a non-degenerate up-cone, i.e. one that contains a non-empty proper up-cone. In the current context of use, the up-cone may be degenerate in fact. It may have an empty extension in virtue of the prior outright concession that the speaker-preferred item in the cone is ruled

\textsuperscript{108}Essentially: we will see that one of the binary parameters distinguishing typical claims from typical concessions, namely initiative, is inverted. This is what we indicate by specifying that $S$ is a counterclaim.

\textsuperscript{109}For this reason we have an inversion of the initiative assignment which goes with prototypical claims (speaker's initiative) and denials (addressee’s initiative). A claim normally initiates a debate, and a concession is a response to it. Cf. Merin (1994).
out. (If we take the minimal case of a two-element up-cone, say the ordered set (‘She killed him’, ‘She hurt him’). But, save for that prior concession or admission, at least retains the characteristic role which distinguishes, for instance, At least x cats are cool from plain x cats are cool where x is a numeral or a determiner denoting a less specific portion quantifier of Lindström’s type <1,1> such as some or a few. The up-cone may be factually degenerate, but it is counterfactually non-degenerate. The counterfactual world here is that of the speaker’s consistent but presently unfulfilled wishes. This latter fact is reflected in an expressive possibility which is designed to mitigate the counterfactuality of the construction and the attendant resignation which the speaker so clearly conveys. A less defensive, less submissive utterer can suspend the concession of non-instantiation of the preferred eventuality. In our examples this could be done by means of the alternative replies, which we might dub (67b’) and (68a’), in the following dialogue variants:

(69) a. Alpha: She hurt him.
   b. Beta: At least she didn’t kill him, if indeed she (did as much as) hurt him (at all).

(70) b. Beta: She didn’t kill him.
   a. Alpha: At least she hurt him, if indeed she didn’t kill him (after all).

Note that, given the suspender clauses in any of their forms, we can readily omit At least from (67b) and (68a) while retaining all the information about the structure of interests and acts which it afforded.

Next consider a sentence involving numerals. Suppose the CCB’s hitwoman is competing in the CCB’s internal Golf Championship. Golf is a pastime, success at which is monotone decreasing in the number of strokes expended in putting the ball in the hole. For an 18-hole course, the theoretical minimum

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110 Could we conclude, in support of the ordinal HS doctrine, that ad-determinant at least X has for its meaning only an intimation (call it implicature, or whatever), that the proper up-cone of X is non-empty and therefore not ruled out by Scalar Implicature? If so, we should have to treat at most, for which this account will not work in a basically truth-conditional framework, in a radically different manner. This is too high a price to pay, unless we give up explanatory reliance on truth conditions altogether.

111 The association of positive preference and non-empty proper up-cone (albeit counterfactual) is retained in counterfactual and then invariably disparaging use of adsentential at least. Alpha says: She didn’t kill him; she didn’t even touch him. Beta replies, cohesively: At least she could have hurt him. However, there is no room for collision, nor for consolation, when Beta’s pronoun is second person, and when Alpha is replaced by a remorse-ridden Hitwoman whose subject pronouns are first person.

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The number of strokes attainable while completing the course is 18, one per hole. This minimum is never attained on a serious course. Suppose now that Alpha and Beta are doing a postmortem on Hitwoman's performance at the tenth hole of the CCB course, which is a serious course.

(71) a. Alpha:  She didn't card three.
   b. Beta:  She carded five, if (indeed it took her) that many (strokes).

To card \( n \), the term coming from golfing language, as I understand from Horn (1972/76), means to perform a temporally ordered set of strokes whose last element lands the ball in the hole and whose cardinality is \( n \). Here is a plausible story to go with the discourse. Alpha, whose own favourite for eventual promotion to chief executive is Hitgirl, is felt to be casting aspersions on Hitwoman. Not to have carded three, in (71a), means doing worse than having carded three, i.e. having carded more than three. Beta is defending Hitwoman, albeit in a guarded way consistent with having to rely upon medium grade human intelligence of the tournament's outcome. What (71b) means is that Hitwoman carded at most five and perhaps did better, i.e. carded less than five.

Clearly, Alpha's attack on Hitwoman's golfing prowess (the extent of prowess being a reliable indicator of executive potential) leaves open the possibility that Hitwoman carded 4, 5 or yet more strokes. On the conservative analysis of negation as complementation, we infer that the sentence meaning of three in the corresponding unnegated sentence She carded three, perhaps asserted by a careless assistant to Alpha, would have claimed 'at most 3', because (71a) claims 'more than 3'. This hunch is confirmed spectacularly by Beta, whose generous concession of five (a claim based on hearsay evidence) is clearly to be understood as implying 'at most five'.

Consider, then, a variant of dialogue (71), namely

(72) a. Alpha:  She didn't card three.
   b. Beta:  At least she carded five (if indeed it took her that many strokes).

Here Beta's diffident defence (72b) of Hitwoman tells even the person who knows nothing at all about golf that carding a lower number is more desirable than carding a higher number. We can even omit the parenthesized suspender tag-on. (72b) also tells us that Hitwoman did no worse, i.e. did nothing less in evaluative terms. And 'nothing less than' here means 'at least' in much the way

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\(^{112}\) The etymology of this denominal verb is from the piece of stiff paper used for writing the score on.
that ‘no less than’ does for subsentential arguments.\footnote{If Hitwoman and Hitgirl were competing on the CCB rifle range or in any other pursuit where success was monotone increasing in cardinality of a point set, the intuitive fit would be even more obvious, though less revealing. This kind of example would, moreover, simply be a numerical variant of our first scenario.} Again we find the less-than-best intimation associated with the non-degenerate potential up-cone. And once again we have the interpretive possibility of an adversarial context of use supplemented by one that is no less salient. This possibility is one of consolatory discourse among allies who have, both of them, been supporting Hitwoman in the race for the top job.

To sum up the positive message of this little essay on At least: We see reflected in our data the relation between the sentence scope At least and its subsentential variant. Each is by default associated with a claim (of which counterclaim is a subvariety), the act type typically associated with an up-cone. Each requires there to be a non-empty proper up-cone, either open for realization or else unrealized as a matter of brute fact (adsentential). The Gross Claim Theorem of Appendix 1, which is part of Theorem 1 in Section 4, spells out the implications of this requirement for prima facie relevance structure.

The less-than-best actualized reading of the adsentential contrasts with the less-than-best potentiality reading of the ad-determinant. Something better than the generating point of the up-cone is still deemed possible in the latter case. Both less-than-best effects and the phenomenology of suspender tag-on clauses reveal a tight correlation between discourse participants’ ostensible preferences and the ostensible incentives which they must back their demands with. Such incentives are often in scarce supply, and suspenders, like the expressive device ‘at least’, are tools for making the best of this predicament. We recall here that, in the domain of ordinary indicatives, incentives take the form of evidence, and demands are assertions, i.e. epistemic claims, advanced to ends that usually take us out of the domain of pure knowledge and belief.

Thus, we could marvel at the phenomenology of these very simple discourse schemata. Even when their content vocabulary is rolled back to anodyne near-emptiness, or replaced by nonsense syllables, they engage surprising riches of sociopolitical and emotive relations. The logical empiricist will suspect that this area of meaning poses the real challenge for a decision-theoretic semantics; but also that there are, at present, few contenders of any description for taking it up.\footnote{The present paper is No. 110 in the series Forschungsberichte der DFG-Forschergruppe ‘Logik in der Philosophie’, University of Konstanz, 2003; and part of the reader for the course ‘Relevance and Decision-Theoretic Semantics’, ESSLLI03 Summer School, Vienna, August 2003.}
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