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Possible Worlds in Model-Theoretic Semantics:
A Linguistic Perspective

I. Historical Background

Possible worlds came into linguistic semantics from philosophy and logic. While
the notion of possible worlds goes back at least to Leibniz, the foundations for
contemporary possible worlds semantics were laid in the late 1950's and early
Alfred Tarski (1935) introduced the related notion of alternative models in the
semantic explication of the notions of logical entailment and logical validity. On
Tarski's account a sentence $\phi$ entails a sentence $\psi$ if $\psi$ is true in every model in
which $\phi$ is true; and the sentence $\phi$ is valid, or logically true, if it is true in all
possible models. Carnap (1947) came close to the notion of possible worlds with
his state-descriptions, which nevertheless are crucially different from possible
worlds in that state-descriptions are sets of sentences, i.e. linguistic objects,
whereas the possible worlds of possible-world semantics are parts of the model
structures in terms of which languages are interpreted. Quoting Carnap,

"A class of sentences in $S_n$ which contains for every atomic sentence either this sentence or its
negation, but not both, and no other sentences, is called a state-description in $S_n$ because it
obviously gives a complete description of a possible state of the universe of individuals with
respect to all properties and relations expressed by predicates of the system. Thus the state-
descriptions represent Leibniz' possible worlds or Wittgenstein's possible states of affairs." (p. 9)

Carnap systematized the intension-extension distinction, analyzing sameness of
intension as sameness of extension in all state-descriptions, and linking the
semantics of modal operators like "necessarily" and "possibly" to the intensions
of the expressions they apply to. As Hintikka notes in his discussion of Carnap's
contributions to the development of possible-worlds semantics, however, Car-
nap did not take the retrospectively natural further step of uniformly analyzing
intensions as functions from possible worlds (or state-descriptions) to the
corresponding extensions, as Montague later did.

Because of the ways in which state-descriptions differ from later conceptions
of possible worlds, Carnap's approach applied smoothly only to the purely
logical modalities, not to other philosophically important modal notions such as
physical necessity or moral obligation. Indeed, even now the different notions of
alternative models, of state-descriptions, and of possible worlds are not always
clearly distinguished. For some purposes, any one of the notions (or several others) can be made to serve equally well, but for other purposes the distinctions are crucial. The conflation is particularly easy to make, and likely to be benign, for purely extensional languages on finite domains, where the possibility that a certain predicate might have had a different extension can be taken to reflect either different ways the world might have been or different meanings the words in the language might have had.

It was Kanger (1957) and Kripke (1959), (1963) who first emphasized the importance of distinguishing the notion of possible worlds from possible models and added possible worlds into the models as part of the semantic interpretation of various modal notions, particularly necessity and possibility. In the interpretation of modal propositional logic, which is basically just like propositional logic plus a necessity operator, written $\Box$, each model $M$ will consist of a set $W$ of possible worlds, an accessibility relation $R$ between worlds, and an assignment of truth values to each atomic sentence in each possible world. Necessity is then interpreted as truth in all accessible possible worlds in the given model. So the basic clause for a formula $\Box \phi$ is given as follows: $\Box \phi$ is true with respect to $M,w$ if and only if $\phi$ is true with respect to $M,w'$ for all $w'$ such that $wRw'$ ("$w'$ is accessible to $w"$). Or in the notation which we will use in discussing semantics in this paper, where $[\alpha]$ indicates the semantic value of an expression $\alpha$ and "1" represents the truth-value "True", the clause above would be written as follows: $[\Box \phi]_M^w = 1$ iif $[\phi]_M^{w'} = 1$ for all $w'$ s.t. $wRw'$.

With this enrichment of the models Kripke and Kanger were able to show how different axiomatizations of modal logic corresponded to different accessibility relations among the possible worlds and hence how different competing accounts of the properties of necessity and possibility could all represent different reasonable notions that might correspond to different kinds of necessity such as logical necessity, deontic necessity, metaphysical necessity, etc. under different views about the nature of the possibilities involved. For example the axiom $\Box \phi \rightarrow \phi$, corresponds to the requirement that the accessibility relation be reflexive, i.e. that if $wRw'$, then $wRw$. Hintikka (1962) showed the value of possible worlds for doxastic and epistemic logic with his well-known treatments of knowledge and belief. (Actually, he used "model sets" in his earliest work, possible worlds later.) The basic idea is to conceive of a certain agent’s knowledge in a given possible world as dividing up the set of possible worlds into those compatible with what he knows in the given possible world and those that are not; similarly for belief. Again various axiomatizations of the notions of knowledge and belief can correspond to various accessibility relations that serve to define which are the agent’s belief worlds from the perspective of a given possible world. So for example if we represent the sentence "a believes that $\phi" as $B_a \phi$, we can indicate Hintikka's

analysis of the semantics of such a sentence as follows: $[B_a \phi]_M^w = 1$ iff $[\phi]_M^{w'} = 1$ for all $w'$ such that $wRw'$ i.e. for all $w'$ such that $w'$ is compatible with $w$. And similarly for knowledge, using an operator $K_a$. Both of these notions are also axiomatizable in various ways, analogous to modal logic. There are current applications of Hintikka-type analyses of knowledge and belief in the study of the logic of distributed systems in computer systems, consisting of multiple separate processors linked by potentially fallible communication channels.

Montague contributed crucially to the development of formal semantics with his development of intensional logic, his combination of pragmatics with intensional logic (Montague 1968, 1970a), and his applications of this work to natural language (Montague 1970b, 1970c, 1973), which gave rise to so-called "Montague Grammar". The intensional logic which he developed unified modal logic, tense logic and the logic of the propositional attitudes; pragmatic aspects were introduced to treat the indexical character of such words as now, I, and here (this latter development represents joint ideas of Montague, Dana Scott, and Hans Kamp).

Starting from the representation of propositions as functions from possible worlds to truth-values, properties as functions from possible worlds to sets, individual concepts as functions from possible worlds to individuals, Montague generalized these notions to a rich type theory with expressions of intensional types denoting functions from possible worlds to values of the corresponding extensional types for a broad range of types. The combination of a rich type theory, pervasive intensionality, and rich pragmatic features (particularly concerning indexicality) gave the languages that Montague developed a very rich expressive power. With his papers, Montague (1970b, 1970c, 1973), he articulated a general theory of syntax and semantics for both natural and formal languages and showed with several sample fragments how it might be applied to English as well as to the semantics of the intensional logic itself.

One important factor in understanding why Montague’s work made such an impact on linguistics is that Montague offered not just a richly expressive language such that one might be able to express in it every proposition expressible in English – for that matter, given a rich enough ontology, one could probably express everything expressible in Montague’s intensional logic in first order predicate logic. What Montague also did and no previous logician or philosopher had done was to provide an explicit mapping from a reasonable English syntax into that semantics, embedded in a general theory treating syntax and semantics as algebras and requiring that the mapping between them be a homomorphism. That constraint can be viewed as a particularly strong version of the compositionality constraint, often referred to as Frege’s principle, which states that the meaning of a complex expression is a function of the meanings of its parts and of their syntactic mode of combination (see Partee 1984).

David Lewis, among his many important contributions to the development of possible worlds semantics and related issues in philosophy of language and
metaphysics, showed in his work on convention (Lewis 1969, 1975) how one could distinguish the purely formal specification of a language from the empirical question of which language is the language of a given language community. He and Robert Stalnaker also showed the fruitfulness of possible worlds in the analysis of counterfactual conditionals (Lewis 1973, Stalnaker 1968): "If p had been the case, then q would have been the case", or as Lewis represents it, "p □➔ q". For the Lewis-Stalnaker analysis, assume we have a measure of similarity among possible worlds so that we can speak of w being closer to w than w" is: this relation is vague, but so are counterfactuals, arguably in the same way. Then the counterfactual above is analyzed as: p □➔ q ≡ w ∗ w ∗ = 1 if g w ∗ = 1 for that world w" which is the closest world to w such that g w ∗ = 1. Thus, the counterfactual is true in the actual world if q is true in that world which is as much like the actual world as possible but with p true. Lewis was also influential with his methodology: "In order to say what a meaning is, we may first ask what a meaning does, and then find something that does that" (Lewis 1970). Linguists working on semantics prior to Lewis and Montague had been working with what in hindsight could be seen as uninterpreted representational languages of unknown expressive power, often ad hoc modifications of first-order predicate logic, and not surprisingly had had at least as much difficulty in articulating what their formalisms were supposed to mean and what constraints there were on them as logicians had had in articulating the semantics of their languages before the advent of model theory. Lewis helped to clarify and extend the notion of meanings as functions, and showed how Montague's strategy for dealing with tense, modality, and pronouns might naturally be expected to extend to meanings as functions not just from possible worlds, times and speakers but from some generalized index containing all the parameters by which extension might vary, such as demonstrated objects (for the demonstratives this and that), degree of precision, degree of salience of various items in the context of discourse, etc.

Cresswell (1973) later argued persuasively against Lewis's view of an index as a discrete index a “n-tuple” and in favor of a more generalized context parameter, arguing that there is no limit in principle to the aspects of context that may be relevant to interpretation. I believe that most linguists are inclined to agree with Cresswell. Cresswell also contributed a great many specific analyses of English constructions, including work on such thorny problems as the semantics of comparatives, of adverbial prepositional phrases and the mechanism of point-of-view in expressions of location and motion, of mass terms, of tense and aspect and its interaction with quantifier scope, and most tenaciously on the semantics of propositional attitude reports, with his most recent views rejecting a pure possible worlds approach. (I will come back to the topic of propositional attitudes below.)

Cresswell and Lewis were both concerned with the metaphysical foundations of possible worlds theory at least as much as with its application to semantics (I will return to the foundational questions in section IV.) With the work of Montague, Lewis, Cresswell, Thomason (1976) and others it began to be clear that intensionality, which in their theories was analyzed in terms of possible worlds, was the norm rather than the exception in natural languages. Let me illustrate what this means before going on with the historical sketch.

Following Carnap (1956), we call a grammatical construction extensional, relative to a certain syntactic and semantic analysis, if the extension of the whole is a function of the extensions of the parts. And we will say that a grammatical construction is intensional, relative to a certain syntactic and semantic analysis, if the extension of the whole is a function of the intensions of one or more parts and the extensions of the remaining parts. Following the standard treatment of possible-worlds semantics, we take the extension of a sentence to be a truth value, and its intension to be a function from possible worlds to truth values (the intension of a sentence is called a proposition); the extension of a one-place predicate is taken to be a set of individuals, and its intension, called a property, is taken to be a function from possible worlds to sets of individuals. The matter of term phrases is made more complicated by the differences among proper names, definite and indefinite descriptions, and quantifier phrases, but for the simplest cases we can say that the extension of a term phrase is an individual and its intension is a function from possible worlds to individuals, often called an individual concept. (Cresswell coined the term hyperintensional for constructions where not even the intensions and extensions of the parts suffice to determine the extension of the whole; whether any natural language constructions are in fact hyperintensional depends primarily on the correct analysis of propositional attitude sentences. We will assume that all natural language constructions are either extensional or intensional until we explicitly take up this question in sections III and IV.)

The clearest and best known test of whether a construction is intensional or extensional is Quine's test for referential opacity (Quine 1960): choose two singular definite descriptions which are contingently coreferential and test whether substituting one for another in a given construction necessarily preserves the extension of that construction: if it does, the construction is exten-

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1 The formulation given here follows Stalnaker's analysis in presupposing the existence of a unique closest world satisfying the given conditions. Lewis's analysis allows that there may be a multiplicity of closest worlds where p is true, and requires that q then be true in all of them for the counterfactual to count as true. I follow Stalnaker here for expository simplicity; nothing here will hang on the difference.

2 An alternative way of viewing this, one more in keeping with Frege (1892) and the treatment of Montague (1973), is to require the extension of the whole always to be a function of the extensions of the parts, and characterize the intensional constructions (Frege's oblique or indirect contexts) as those in which we must assign as the extension of a given expression what would ordinarily (in direct contexts) be its intension.
sional, otherwise intensional. While the conclusions to be drawn from "failure of substitutivity" in such cases are of course somewhat theory-dependent, the following examples can at least plausibly be diagnosed as indicated (Partee 1974).

1) Verb plus direct object (\(\text{VP} \rightarrow \text{NP}\)) can be intensional; the classification depends on the verb. We give an example of each.

1.1) *Extensional: strike + NP*

Test:
(i) The bullet strikes the owner of the red Volvo.
(ii) The owner of the red Volvo is Mary's husband.
(iii) (Therefore) The bullet strikes Mary's husband.

Valid, so extensional.

(Note: I am using present tense despite unnaturalness to keep the context of evaluation constant in all three sentences and avoid complications of other tenses.)

1.2) *Intensional: seek + NP*

Test:
(i) John seeks the owner of the red Volvo.
(ii) The owner of the red Volvo is Mary's husband.
(iii) (Therefore) John seeks Mary's husband.

Invalid, (on at least one reading), therefore intensional.

Most transitive verbs are extensional; other intensional ones include want, need, hope for, wish for (and many combinations with for), imagine, picture, offer, owe.

2) Constructions with embedded that-clauses. Verbs and adjectives that take that-clauses as arguments are very often intensional. Tests parallel to those given above would show that all of the following examples involve the intention of the embedded sentence.

2.1) Mary believes that the man in the raincoat is a spy.

2.2) That the bank president works in the bank is obvious.

2.3) It appears that a child has been writing on the walls.

Another frequently cited test for extensionality is the "existential generalization test", illustrated below.

(i) John is sitting on a talking horse.
(Therefore) There is a talking horse such that John is sitting on it.

(ii) John is looking for a talking horse.
(Therefore) There is a talking horse such that John is looking for it.

The validity of (i) and invalidity of (ii) are taken as further evidence for their extensionality and intensionality respectively, supporting the evidence of (1.1) and (1.2) in the text. However, the results of these two tests may not always coincide and I believe it is best as well as more standard to take the substitutivity test as the criterial one.

Among the few purely extensional sentence-embedding adjectives are *true* and *false*, as illustrated in (2.4); the rarity of this case suggests that the construction in general should be regarded as intensional, the extensionality of (2.4) following from the specific lexical meaning of *true*.

2.4) It is true that the bank president is a spy.

3) Verb-phrase adverb plus verb phrase, and sentential adverb plus sentence.

Although it is not always easy to distinguish verb-phrase adverbs from sentential adverbs, one finds in general that sentential adverbs can create referential opacity in the sentences they modify and verb-phrase adverbs do the same in the verb phrases they modify. Examples of intensional sentence adverbs are given in (3.1-3.2), intensional verb-phrase adverbs in (3.3-3.4), extensional adverbs in (3.5-3.6)

3.1) Smith's murderer is probably insane.

(semantic analysis: probably interpreted as a function which applies to the proposition expressed by Smith's murderer is insane.)

3.2) The president of the country is necessarily the commander-in-chief of the armed forces.

3.3) Mary reluctantly bet on the best horse.

3.4) Sam willingly invited the robber into his house.

3.5) Smith's murderer is now insane.

3.6) Mary quickly bet on the best horse.

The fact that almost every English construction includes some instances which are intensional led Montague (1973) to make intensionality the general case, guaranteeing the extensionality of the extensional cases by means of meaning postulates which constrained the models that could be candidates for possible interpretations of English. For instance, a meaning postulate for the verb strike would limit the possible interpretations to those in which the extension in a given world of strike plus its direct object NP would depend only on the extensions of strike and of that NP in that world. For an intensional transitive verb like seek, on the other hand, there would be no such meaning postulate. By relaxing Montague's constraints on the syntactic-semantic mapping slightly, one can also make the distinction between extensional and intensional transitive verbs directly in their type structure, treating the extensional ones extensionally right from the start (Partee and Rooth 1983). The choice between these approaches is an empirical linguistic issue, and the treatment of the intensional cases is the same on both approaches, and in sharp contrast to the "sentential operator" approach to all intensionality, characteristic of treatments which just add certain sentential operators to otherwise first-order predicate logic. Making intensionality the general case, as Montague did, has the linguistic advantage over modified-first-order approaches of letting the semantic structure match natural language surface structure much more closely and hence making natural
languages look semantically much more reasonable than they look if one takes "logical form" to mean a basically first-order-predicate-logic form.

We have already mentioned Montague's steps toward integrating certain aspects of pragmatics with possible worlds semantics. It was natural that the first aspect of pragmatics that he focused on was the treatment of indexicals, since, for instance, the integration of tense logic with modal logic leads directly to the consideration of the indeterminacy of the present tense and of words like now. It is then a small step from there to consideration of I, you, here, and other aspects of the context of a speech act that are directly involved in determining the truth conditions of expressed propositions – in fact in determining what proposition is in fact expressed, given that propositions are identified with their truth conditions on this classical possible worlds approach.

Further attention to pragmatics was shown in the work of Hamblin (1970), David Kaplan (1977) and in the work of Robert Stalnaker (1974, 1976, 1978). Stalnaker introduced the important notion of common ground in a conversation: a set of propositions which all the participants in a conversation take to be shared background knowledge or uncontested background assumptions at a given point in the conversation. The propositions in the common ground in a conversation at any stage determine a set of possible worlds, the context set, which the conversational participants take to be live options at that stage of the conversation, in the sense that as far as the information in the common ground goes, the actual world might be any one of the possible worlds in the context set. To gain information about the actual world, then, is to add propositions to the common ground that serve to eliminate some possible worlds from the context set, the set of live options. On this view, a primary purpose of making assertions (in the factual mode of discourse) is to cut down on the set of possible worlds in the context set, thereby narrowing down the range of uncertainty of the conversational participants as to which possible world is the actual one. Of course, since the common ground may include shared misinformation, there's no guarantee that the actual world is in fact included among those the participants take to be possible at any given stage. In fact, the notion of common ground in a conversation does not in any way actually require that there even be such a thing as an actual world, although it is more difficult to make sense of the purpose of much conversation if it is not taken to be a striving to converge on a more narrowly constrained set of possibilities as to which the actual world may be.

Stalnaker emphasized the dynamic nature of pragmatics in stressing how what is said both depends on the common ground (and other aspects of context) and in turn affects the common ground. His pragmatic analysis of presupposition has been very successful, although much remains to be worked out – this is still a very lively field of inquiry. Stalnaker, building on earlier work of Montague, Kaplan, Kamp and Vlach, also did very important work showing how possible worlds have to be taken to enter the analysis of the interpretation of sentences twice – once as an aspect of context and once as an index at which a sentence is evaluated; this he took to be crucial for understanding the functioning of demonstratives in propositional attitude contexts, among other things.

Extensions of Stalnaker's ideas about pragmatics in the interpretation of a variety of constructions by Kratzer (1978) and Heim (1982, 1983a) led to the suggestion by Heim that the integration of semantics and pragmatics has to be even closer than had been proposed in earlier work. Heim argued that for an explanatory linguistic theory, one wants to take the basic semantic value of a sentence to be not its truth conditions (a function from possible worlds to truth values), as in the classical models, or even a function from contexts of use to a function from possible worlds to truth values, as had been proposed by Stalnaker and by David Kaplan, but rather a function from contexts to contexts, where the notion of context is much like that of Stalnaker's context set, augmented at least by something like "discourse referents" representing the individuals being 'talked about' and available for anaphoric reference at a given point in a discourse. This may seem a small change, perhaps, given that contexts include possible worlds, but it is an important one in emphasizing a much more dynamic view of semantics and a greater inseparability of semantics from pragmatics and creating the potential for greater rapprochement between formal semantics and speech act theory, and incidentally also closer connections with work on the semantics of programming languages, where a dynamic view is common. While on this view the interpretation of a sentence is not simply as a set of possible worlds, possible worlds play as central a role in this picture as in its predecessors. In a theory like Heim's, a sentence is interpreted as a set of possible worlds in the special case where the context set is empty and the sentence itself has neither presuppositions nor contextual effects. The classical possible worlds theory is then viewed from this perspective as a theory applying to such context-independent cases, much like a theory of motion for frictionless planes.

This brings us up approximately to the present in this brief sketch of the development of possible-worlds semantics. In the next section I will describe, also briefly, a sampling of alternative approaches to semantics, including alternatives within a basically possible-worlds tradition, plus non-possible-worlds approaches to semantics that are still within a model-theoretic tradition, and some alternatives that are not model-theoretic at all, so that the non-specialist may have at least a bit of perspective on possible-worlds semantics in relation to rival theories, and the specialist can see clearly where my own biases lie. Then in section III I offer my evaluation of the effects the possible-worlds approach has had on the development of linguistic semantics, and of what features of possible worlds theories have been most crucial in these developments. Finally in section IV, I come back to what I see to be some of the central foundational issues that are being confronted or will soon have to be confronted as the frontiers of research in semantics and pragmatics continue to be pushed back.
II. A Sampling of Alternative Semantic Theories

Semantics is currently the focus of a lot of lively research, much of it interdisciplinary, involving such fields as artificial intelligence, theoretical computer science, information science, and cognitive science, as well as linguistics and philosophy. It is not surprising that there are a lot of competing approaches, rapidly evolving theories, and considerable mutual influence among these diverse approaches. It is hard to draw a road map of the terrain; I can't pretend to be fully representative, but I would like to try to provide at least some perspective on the current scene.

A. Variants of Possible-Worlds Semantics

First, let me mention some differences of approach within the possible-worlds tradition. On the philosophical side, there have been different degrees of metaphysical realism about possible worlds. There are “extreme realists”, David Lewis the best known, who believe not only that other possible worlds do not differ in kind from the actual world, but that each possible world is just as actual from the point of view of its inhabitants as ours is from ours (Lewis 1973). Some philosophers and most linguists prefer to view possible-worlds talk as a useful instrument in theorizing about semantics and other enterprises, while taking a skeptical or an agnostic stand towards the metaphysical reality of possible worlds. Stalnaker in his recent book Inquiry presents a version of “moderate realism” in which there are other possible worlds in the sense of other ways things might have been, but these are not more things of the same kind as the actual world. Rather he takes them to be abstract entities existing in the actual world, “abstract objects whose existence is inferred or abstracted from the activities of rational agents.” (Stalnaker 1984, p. 50.)

While the debates about metaphysical realism do not translate in any simple way to differences in semantic theories, they are directly relevant to differences in broad conceptions of the nature and grounding of a semantic theory of natural language. Since attention to these questions is relatively recent, I will postpone further discussion of this issue to section IV, but note that until quite recently possible-worlds semantics has tended to be dominated by a realist picture such as Lewis’s, creating some tension for linguists attracted by the technical merits of possible-worlds semantics but with strong leanings (derived from the Chomskyan perspective) towards a psychologistic basis for any branch of linguistic theory.

A related difference within possible-worlds theories is whether possible worlds are taken to be primitive or constructed. An extreme realist will generally, perhaps necessarily, take possible worlds to be primitive, but a moderate realist or instrumentalist has a choice. Some conceive of possible worlds as built up out of individuals and properties – that then leads to questions about the nature of individuals and the nature of properties, questions on which there is at least as much debate as there is about possible worlds. From the perspective of possible-worlds semantics, one might view possible worlds as analogous to points in a theory of geometry, and simply not make any metaphysical claims as to whether they are to be viewed as primitive, as abstracted from conceptions of three dimensional space, or as possibly otherwise analyzable.

Another dimension of variation, one which linguists tend to concern themselves with more directly, is on theories of syntax and of the relation of syntax to semantics and pragmatics. While most of these differences are beyond the scope of this paper, there are some that should be mentioned here. One important point is that while not all possible worlds semanticists advocate phrase-structure-like grammars it is probably fair to say that the development of model-theoretic semantics has done much to make the return of near-phrase-structure-grammars possible by providing tools which let one capture relations between sentences semantically in cases where previously the only way to show a systematic relation between two sentence-types was to derive them from a common syntactic source or deep structure via transformations.

Under the heading of different theories of syntax and of the relation of syntax to semantics and pragmatics within the possible-worlds tradition, alternatives to mention include game-theoretic semantics (developed by Hintikka and his colleagues; see Saarinen, ed. (1979)), discourse representation theory (Kamp 1981), generalized phrase structure grammar (Gazdar, Klein, Pullum and Sag 1985), extended versions of categorial grammar (Bach 1984; Dowty 1982; Steedman, and others), head grammar (Pollard 1984), and other theories being developed in the Stanford area using ideas of unification as developed by Marvin Kay and others. Early attempts to wed transformational grammar to possible-worlds semantics (Partee 1975 and others) have generally given way to attempts to use the power of the semantics to eliminate most transformation-like rules from the syntax; in particular there has been a tendency to eliminate essentially all “local transformations” (see Dowty 1978), but often to retain one or another special device for dealing with “unbounded transformations” such as wh-questions or relative clause formation, as well as the representation of scope ambiguities among quantifiers.

One major unresolved point of debate that divides theories within the model-theoretic tradition with respect to the relation between syntax and semantics concerns the existence of a distinct level of “logical form” mediating between natural language syntax and model-theoretic semantics. On Montague’s original theory, the strong compositionality constraint required that any such level be in principle dispensable; the closest analogue to “logical form” in that theory is the “analysis tree” which displays the syntactic derivation of an

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4 On such a view, the “building up” process may also have to be taken as a new primitive, usually modeled via a standard set theoretic approaches treating worlds as maximal consistent sets of propositions.
expression. Since on Montague's theory there is a unique semantic interpretation rule for each syntactic rule, the analysis tree that displays the syntactic derivation of an expression corresponds simultaneously to the steps in the compositional semantic interpretation of the sentence. While it is a practical necessity to employ something like Montague's intensional logic as a metalanguage for representing the resulting model-theoretic interpretation, expressions in the intensional logic have no special status in the theory, and such a representational level is taken to be in principle dispensable. Not all theories in the possible-worlds tradition hold to such a strict version of compositionality, however, and the "realist" tradition in linguistics, discussed more below, leads linguists to continue to investigate the possibility that some sort of intermediate level of logical form or the like might play an important role in the mental representation of semantic interpretations both for individual languages and at the level of semantic universals. Empirical evidence for or against such a hypothesis has been very hard to come by so far, however.

B. Non-Possible-Worlds Theories within Model-Theoretic Semantics

A recent challenge to possible-worlds semantics from within the model-theoretic tradition has been raised by Barwise and Perry with their development of situation semantics (see Barwise and Perry 1983). One of their contentions is that early arguments by Frege, Church, and others to the effect that the extension of a sentence must be a truth-value were flawed, and that the alternative intuition that a sentence describes a reality "local" state-of-affairs can be defended and put on a sound formal footing. They then take as primitive the notion of a situation, corresponding approximately to a limited piece of the actual world; with properties and individuals also taken as primitives, they build up the notion of a situation type. They emphasize the inherent partiality of situations and situation types as opposed to inherently "complete" possible worlds, although it has recently been argued by Stalnaker (ms. 1985a) that there is no obstacle in principle to identifying a situation with the set of all possible worlds that have that situation as a part. (However, problems do seem to arise with the introduction of situations that have other situations as parts, as proposed for the treatment of situations involving perception; see Landman 1985.) Situation semantics has not dealt explicitly with the semantics of the modal constructions such as possibility and necessity which motivated the possible worlds in the first place, and the theory is still very much in flux, so it is probably still too early to say whether the apparent eschewal of the notion of possible worlds represents a substantive difference in the theories or not.

As a converse influence, the attention given to sentences reporting perceptions of events and other local situations within situation semantics is one of the influences that has led many possible worlds theorists to renewed interest in theories which include events along with or in place of times among the primitives in their model structures, as opposed to theories which reduce events to properties of instants or intervals of time as Montague had proposed. The idea that events, processes, and other "situation"-sized entities should be included in models as basic rather than treated reductionistically has surfaced in many places and with many motivations. I don't think one should necessarily see a fundamental opposition between possible situations and possible worlds, however; I think the broad notion of possible worlds, as alternative ways things might have been, is as fundamental in thinking about alternative possible courses of "local" events as in thinking about alternative possible global histories. I thus see the place of situations in a model structure as more closely analogous to that of individuals or events than to that of possible worlds.

C. Non-Model-Theoretic Semantics

One could not do justice to the past and present development of semantics in linguistics without emphasizing the older and still thriving traditions that are not model-theoretic at all. Up until the early 1970's, when Montague's influence began to be felt in linguistics, virtually all linguistic theories of semantics were what we might characterize as representationalist in substance and conceptually in foundation, and in many quarters this is still the dominant trend.

By "representationalist", I mean the view that interpreting a sentence of a natural language is a matter of deriving a representation of it in some suitable language of semantic representation, such as Jerry Fodor's proposed "language of thought" (Fodor 1975, 1981), Katz and Fodor's earlier language of "semantic markers" (Katz and Fodor, 1963), or recent versions of "logical form" advocated by May (1977, 1985), Higginbotham (1983), or others in Chomsky's "GB" framework (Chomsky 1981). The "conceptual dependency" representations or "semantic networks" used in some current artificial intelligence work are close relatives of this approach to semantics. Against the charge that just translating from one uninterpreted language to another gets one no closer to the first thing about the meaning of a sentence (a charge articulated by David Lewis (1970)), namely its truth-conditions, the responses are usually twofold. (i) If the semantic representation language is universal, as is often posited, so that the sentences of any natural language can be represented in it, then it can satisfy at least one goal of semantics, namely to provide a theoretical framework for studying how languages of the world with different syntactic structures can express the same meanings, and to provide a language-independent way of

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3 Another way of putting the matter would be to say that any candidate level of "logical form" in strict Montague grammar must either be dispensable (like Montague's language of intensional logic) or must count as part of the syntax (like Montague's syntactic rules for dealing with quantifier scope).

4 Raymond Turner (personal communication) points out that one can question the special status accorded on this view to Zermelo-Frankel set theory, the meta-theory for the underlying model theory, and it might be argued that the language of ZF set theory is less well suited than intensional logic as a metalanguage for semantics.
characterizing these meanings (at least independent of any particular natural language.) (ii) Furthermore, it is still a matter of debate whether the way speakers of natural languages use semantics in speaking, understanding, and carrying out inferences is better modeled by model-theoretic or proof-theoretic means or some combination of the two. A strictly model-theoretic approach with no intermediate representational level is committed to denying that semantic competence involves in any systematic way the syntactic manipulation of expressions in some language of semantic representation, while a strictly representationalist account is committed to that being all there is to semantic competence, denying any systematic role to intrinsically model-theoretic operations. However, against this response it can be replied that model-theoretic approaches normally are accompanied by one or more representational languages, often with a proof theory as well as a model theory, while representationalist proposals often have neither. In the absence of an actual proof theory or other grounding, the import of proposed representational languages is difficult to judge.

Actually, this characterization of representationalism reflects not only the representationalist position but the conceptualist position as well, by which I mean the position that as far as the semantics of natural languages goes, criteria of adequacy of a given theory derive ultimately from the attempt to account for what is “in the head” of the competent native speaker of a language. This position, a basically Chomskyan one, is directly at odds with the Fregean anti-psychologism inherited by much work in logic and formal semantics in philosophy. (See Partee (1979), (1980)) While representationalism and conceptualism often go together, particularly in the Chomskyan tradition, they are not inherently inseparable; and one could mention recent Katz (1981), as an example of a non-conceptualist representationalist, and Johnson-Laird (1983), as an example of a conceptualist who advocates a model-theoretic approach and has been seeking plausibly psychologically realizable versions of possible-world semantics.

Conceptualism in linguistics is also related, I believe, to functionalism in the philosophy of mind and its influence on the foundations of cognitive science. For reasons that may be partly a matter of historical accident, cognitive science as it has been developing in the U.S. in the last decade or so has been dominated by a very syntactic “information-processing” model of semantics, influenced heavily by the computational metaphor, and it is only recently that new interest in the foundations of the semantics of computer languages has begun to bring serious interest in possible worlds semantics and other model-theoretic approaches to semantics into mainstream cognitive science. To put it very crudely, one might say that the position of conceptualists-functionalists is that all that really matters for semantics is what the language user thinks the sentence means and that in turn can be characterized by the state of his mind viewed as a machine with a lot of different internal states functionally related to each other; then, equally crudely, one could characterize the new interest in the semantics of knowledge representation languages used in artificial intelligence research as a concern with the question of how one can attribute informational content to internal states of a computer. It is no accident that this is in a sense a revival of the basic problem of intentionality – what it is for one mental state to have informational content. What is more surprising is that it has seemed possible to avoid coming to grips with that problem for this long in much of the forefront of research in cognitive science. The connections between possible worlds semantics and representationalist theories are bound to become much closer and much more intertwined in the coming years, I believe.

III. The Value of Possible Worlds for Linguistic Semantics

As I tried to emphasize earlier, the value of possible worlds for linguistic semantics is largely independent of metaphysical issues. Linguists, qua linguists, tend to be instrumentalists about metaphysics, in the sense that they will tend to judge competing foundational theories more by their fruitfulness in helping to lead to insightful explanations of linguistic phenomena than by any other kinds of arguments. (In fact this is sometimes carried so far that not all linguists seem to believe that a theory of the semantics of natural language needs to be consistent – although I think that in some cases this reflects confusion over where to deal with the fact that language users can have inconsistent beliefs.) So for example, in the case of questions about the nature of time – whether it is discrete or continuous, whether there is a first or a last moment, whether it is linearly ordered across possible worlds or whether one should rather posit a system with, say, “branching futures” but “linear pasts” – the primary concern of the working linguist is which set of assumptions will enable her to give the most satisfactory account of the semantics of tense, aspect and temporal adverbials in a given language and across languages. In fact, as Bach (to appear) has suggested, if one were to find evidence that different underlying assumptions of this sort gave optimal semantic accounts for different natural languages, one would have an important contribution to an enterprise he has dubbed “natural language metaphysics”, possibly giving new respectability to the kinds of questions raised by Sapir and Whorf earlier in this century about the relation between language and “world view”.

I should also mention at the outset of this section that for the first decade or so of Montague Grammar, roughly the 70’s, linguists working in Montague’s possible worlds framework did not feel competent to tamper with the foundations, and tended to take the whole package of PTQ or at least Montague’s Intensional Logic as a unit – including the type theory, the single set of possible worlds per model, linear time across all possible worlds, classical logic, functions that were always total, a single domain of possible entities across possible worlds, intensions abstracting simultaneously over worlds and times, a typed rather than an untyped λ-calculus, etc. There was even a tendency to regard
“model-theoretic semantics” and “possible-worlds semantics” as equivalent, and both as equivalent to Montague Grammar. Closer cooperation and mutual influence among linguists, philosophers and logicians has led to a much greater proliferation of theories within this general approach in recent years. So in assessing the value of possible worlds for linguistic semantics, it is not always easy to separate out the contribution of possible worlds proper from other aspects of possible-worlds semantics as developed by Montague and others.

I would summarize the kinds of value possible-worlds theories have had for linguistic semantics as being of two kinds: (i) possible worlds as a technical tool have helped to provide an appropriate structure on the space of meanings; I will illustrate what I mean by that with examples below. (ii) As a further benefit, not so much within linguistic proper but of potential value for linking linguistics to other disciplines, the possible-worlds conception is of great help in relating linguistic meaning to other kinds of informational content, and I want to say a little bit about that before getting into specifically linguistic examples.

As emphasized by Stalnaker (1984), possible worlds help to provide a general account of the intentionality of mental states that does not make all intensionality irreducibly linguistic. The reader may have noted the occurrence of both the words ‘intentionality’ and ‘intensionality’, and it is at this point that I should make the distinction explicit. ‘Intentionality’ as used in the contemporary philosophical literature can be said to be concerned primarily with the question of what it is for mental states to have informational content – sometimes expressed as the problem of ‘aboutness’. ‘Intension’ is a technical term used in opposition to ‘extension’, analyzed in different ways in different theories. The term in its current usage has its roots in Carnap (1947) (although the term is older); Carnap suggested that individual terms, predicates, and sentences each had an intension as well as an extension, identifying these as follows:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Intension</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>sentence</td>
<td>proposition</td>
<td>truth-value</td>
</tr>
<tr>
<td>predicate</td>
<td>property</td>
<td>set</td>
</tr>
<tr>
<td>individual term</td>
<td>individual concept</td>
<td>individual</td>
</tr>
</tbody>
</table>

Montague unified and generalized Carnap’s intension-extension distinction, developing a typed intensional logic in which intensions are uniformly analyzed as functions from possible worlds to corresponding extensions. The link between “intension” and “intension” can be argued to be the notion of possible world: both linguistic content and the content of mental states such as believing or perceiving involve discriminating among different possible worlds. To be capable of intensional states is on this view to be capable of in some sense entertaining different possibilities and in some sense discriminating among them. Such discrimination does not in principle require language nor does it require that there be some single absolute set of all possible worlds involved whenever we talk about discriminating among alternatives. It thus gives one a good handle on talking about the similarities and the differences between e.g. the content of an act of visual perception and the content of an uttered sentence; and also the similarities and differences between the content of the mental state of a dog which remembers where it buried a certain bone and its master who also knows where the bone is buried – an oft-cited case in which one problem is how to account for the sense in which they may be said to know the same thing without attributing to the dog a full set of mental states fully isomorphic to those of its master nor a sentence in the same “language of thought” as its master presumably thinks in. Even if the “language of thought” approach characteristic of representationalist views of semantics were able to match the model-theoretic approach in handling the kinds of linguistic examples discussed below, it might have to be at least supplemented by something like the possible-worlds approach in order to make appropriate connections between linguistic content and other obviously related cases of intentionality and content such as those just mentioned.

Let us turn now to some specific examples of linguistic phenomena where possible worlds have played a role in the analysis, and try to flesh out the claim I made above that possible worlds help to provide an appropriate structure on the space of meanings.

(i) Let us first consider the identification of a proposition with a set of possible worlds, that is with the set of possible worlds in which it is true. This is equivalent to identifying a proposition with its truth conditions or with a function from possible worlds to truth values, or with a method of discriminating among a set of possibilities. Ignoring the additional complications relating to pragmatics and context-dependence that I mentioned earlier, a proposition in this sense is taken to be the meaning of a sentence on the classical possible-worlds view. A first point to note about structure is that on this analysis, one can ascribe various logical relations to propositions without any necessary point-for-point corresponding ascription of “logical form” to the sentences that express them. For example, consider sentences (1)–(4) below.

1. John and Bill are mortal.
2. John is mortal and Bill is mortal.
3. John is mortal.
4. Bill is mortal.

In terms of overt syntactic form, (2) is a conjunction of (3) and (4), while (1) is not. Semantically, since (1) and (2) express the same proposition, (1) is just as much the conjunction of (3) and (4) as (2) is; on the possible-worlds account this is straightforward, since (1) and (2) pick out exactly the same set of possible worlds, a set which is the intersection of the sets of possible worlds picked out by (3) and (4). In a typical model-theoretic account such as that provided by Montague, (1) and (2) can be given different syntactic structures, analysis trees isomorphic to their respective surface structures, and these distinct syntactic structures are interpreted by distinct semantic rules: rules for noun phrase conjunction in the case of (1) and for sentence conjunction in the case of (2). If
the rules are correctly formulated, they lead to the same truth-conditions, though via different compositional routes. On a purely model-theoretic account, there need be no “level” at which (1) and (2) have the same “semantic representation” or “logical form”. To follow an analogy suggested by Stalnaker, in ordinary arithmetic the different expressions “7−2” and “1+4” determine the same number, but that need not mean that the number so determined is intrinsically a sum or intrinsically a difference, nor that the number 5 should be viewed as having the numbers 4 and 1 as “parts” just because it is their sum. Linguists originally tended to take a very syntactic view of semantics, focussing on the obvious relation of an overt sentential conjunction like (2) to its syntactic parts, (3) and (4), and tending to want to reduce the syntactically less obvious case of conjunction in (1) to that of (2). The difference in the two conceptions of conjunction, to summarize, is this: for a sentence A in a “logical form” to be the conjunction of two sentences B and C, A must contain B and C as parts with a conjunction sign between them; but for a proposition A (a set of worlds) to be a conjunction of propositions B and C, A must be the intersection of B and C; and this latter does not require that B and C be in any sense “parts” of A. Thus the possible worlds account, but not a “logical form” account, can account directly for the fact that (1) expresses the conjunction of (3) and (4) just as much as (2) does. Propositions viewed as sets of possible worlds have a Boolean structure with respect to the Boolean connectives and, or, and not.

The case of entailment is even clearer than the case of conjunction. In terms of possible worlds a proposition A entails a proposition B if the set of worlds in which A is true is a subset of the set of worlds in which B is true. In order to get a corresponding notion in a theory of logical form, one needs to articulate a set of axioms and of rules of inference and define a notion of consequence in terms of proof in such a system. Possible worlds semantics has made it possible to say a great deal about entailment relations among sentences of natural languages without having to enter into questions of choices among possible axiomatizations of natural language inference systems, choices which would probably be extremely difficult to make on principled grounds if possible at all.

(ii) As mentioned earlier, the possible worlds analysis of intensionality has made it possible to appreciate and analyze the rampant intensionality of natural languages, especially by virtue of Montague’s use of function-argument structure to interpret most basic syntactic constructions. The idea that for any possible extensional type of interpretation there is a corresponding intensional type has been a fruitful one. So, for instance, old puzzles about the behavior of “role-denoting” definite descriptions like the president can in many cases be analyzed by assuming the expression may denote a non-rigid individual concept, whose scope with respect to other operators in the sentence may be ambiguous. Example (5) is ambiguous in this way.

(5) The president is gaining power.

Likewise, some old puzzles in the theory of actions can be seen to involve action-descriptions under the scope of intensional adverbs in cases where the descriptions have different intensions but identical extensions. In this manner the invalidity of an inference like that in (6) can be explained.

(6) John raised his arm intentionally.
   In raising his arm, John set off the alarm.
   Therefore John set off the alarm intentionally.  (INVALID)

Similar analyses have elucidated the semantics of intensional adjectives like good, skillful, strict and brought to light the hitherto unsuspected intensionality of the English progressive aspect, not to mention the expected applications to the semantics of modals, sentential adverbs, and many sentence-embedding constructions. In turn, as the limits of what can be explained by intensionality have been reached, the role of pragmatic factors in context-dependent interpretation has become clearer and the magnitude of the difficulty of what is probably the hardest problem in semantics, the interpretation of propositional attitude constructions such as belief-sentences, has become more vividly apparent.

(iii) The semantics of belief-sentences and other propositional attitude constructions has been an interesting battleground for possible-worlds semantics. Hintikka’s use of possible worlds to elucidate the analysis of knowledge and belief (Hintikka 1962) was an important advance in clarifying many of the issues around referential opacity in the context of belief-reports, distinctions between knowledge by description and knowledge by acquaintance, etc. But the very kinds of logical relations among propositions that I talked about as an advantage of the structure of possible worlds semantics in (i) above has been argued to be an insurmountable obstacle to an adequate treatment of propositional attitudes in terms of possible-worlds semantics. Let me first sketch the standard treatment of propositional attitude sentences in possible-worlds semantics before describing the problem.

Consider sentences (7)–(9).

(7) Martha believes that the president of the bank is honest.
(8) The president of the bank is the man who bribed the judge.
(9) Martha believes that the man who bribed the judge is honest.

Clearly (9) does not follow from (7) and (8). This can be accounted for straightforwardly on the assumption that believe combines with the proposition expressed by the sentence in the that-clause, i.e. with the whole intension of the sentence. Then the intensions and not merely the extensions of the parts can affect the truth-value of the whole containing sentence.

Sentence (8) asserts that the two definite descriptions have the same extension, but clearly their intensions can differ. And since the intensions matter, (9) does not follow from (7) and (8). This account can be seen to be essentially the
same as that proposed by Frege (1892): Montague took his work as formalizing and extending Frege’s insights.)

So ordinary referential opacity in belief-contexts of the sort discussed by Quine and others is handled in a possible worlds semantics in basically the same way as other modalities — in any intensional context, whether created by an adverb, a modal verb, a propositional attitude verb, or whatever, substitution of one expression for another with merely the same extension will not be expected to preserve truth.

However, substitution of one expression for another with the same intension will be guaranteed to preserve truth in any intensional context, since in a possible world semantics, sameness of intension amounts to semantic identity. And herein lies the putative problem for the treatment of propositional attitudes, since the available data seems to give at least strong presumptive evidence that the substitution of co-intensional expressions in propositional attitudes contexts does not always preserve truth. That is, we can in our ordinary sincere belief-attributions to be willing to ascribe to an agent belief in some proposition p without thereby ascribing to that agent belief in every proposition logically equivalent to p. Yet if propositions are analyzed as sets of possible worlds, all logically equivalent propositions pick out the very same set of possible worlds, so there’s no way to interpret a sentence as ascribing to someone a belief in one proposition without thereby ascribing belief in the whole logical-equivalence family.

There have been many reactions to this problem, and there is no space to do them justice here besides, I suspect this issue will be a prominent one in the discussion of possible worlds in philosophy in this symposium, since it is a problem that has traditionally received more attention from philosophers than from linguists. Responses have ranged from outright rejection of the whole possible-worlds approach (but without, to my mind, any better account of propositional attitude reports being available in any other known framework), to enrichment with quotation-like devices to allow more fine-grained distinctions than can be gotten with possible worlds alone, to defense of the possible-worlds account of propositional attitude attributes with a bit of linguistic reanalysis of the sentences that seem to cause the trouble. Most recently there have begun to be attempts to combine aspects of representationalist accounts with aspects of possible-worlds model-theoretic accounts, as in discourse representation theory and some of the suggestions made within situation semantics.

(iv) One of the earliest and most striking technical successes of possible worlds semantics was in the analysis of counterfactual conditionals of David Lewis (1973) and Robert Stalnaker (1968). This was mentioned briefly in section I: recall the semantic analysis given there, repeated below as (10).

\[ [p \square \psi]_{M} \equiv 1 \text{ iff } [\psi]_{M} \equiv 1 \]

for that \( w' \) which is the closest world to \( w \) such that \( [p]_{M} \equiv 1 \).

This analysis is of general significance for possible-worlds semantics in that here it matters not just that sentences can get different truth-values in different possible worlds, but that some further structure be able to be imposed on the possible worlds, namely an ordering in terms of similarity to the actual world. This ordering is crucial in explaining some of the classic problems in the logic of counterfactual conditionals, such as the invalidity of “strengthening the antecedent”, illustrated in (11a–d), which can all be true.

\[
\begin{align*}
(11) \, (a) & \text{ If I had struck this match, it would have lit.} \\
(11) \, (b) & \text{ If I had struck this match and it had been wet, it would not have lit.} \\
(11) \, (c) & \text{ If I had struck this match and it had been wet and I had previously coated it with paraffin, it would have lit.} \\
(11) \, (d) & \text{ If I had struck this match and it had been wet and I had previously coated it with paraffin and the paraffin had in the meantime all worn off, it would not have lit.}
\end{align*}
\]

And so on. The possible continuing alternation of positive and negative consequences can be explained by the normal assumption that, for instance, worlds where the match is not wet are more similar to the actual world than the worlds where it is wet; worlds where I didn’t coat it with paraffin are more like the actual world than the worlds where I did; and so on. As long as each successive “strengthening” of the antecedent takes us to worlds further from the actual world than the ones we had previously had to consider, the closest worlds where the new antecedent is true will be different from the closest worlds where the previous antecedent was true, and hence the possibility arises that the consequent will take on a different truth-value there.

More recent work on the semantics of counterfactual conditionals by Kratzer (1981a, b, c) and others suggests that the full story may require having all of possible worlds, propositions, and situations available to appeal to, with each entering into an explanatory account in different ways. It should not be too surprising, however, if counterfactuals stretch the resources of our semantic theories to their limits, given the notorious interrelations among counterfactuals, lawlike statements, generic attributions, and the analysis of possibility and necessity.

(v) The analysis of the semantics of questions and the pragmatics of question-answer relations took a great leap forward with the advent of possible-world semantics, in my estimation, and has continued to develop and flourish. Karttunen (1977) made a very fruitful methodological move when he shifted attention away from the analysis of direct questions to the analysis of embedded questions and framed the problem as one of finding an account of the semantics of embedded questions which could make an appropriate contribution to a compositional semantics for the full range of question-embedding constructions, ranging from \textit{ask who... do to know where... to depend on}, the last of which can take embedded questions for both its subject and its object, as in (12).

\[
(12) \text{ Whether John enjoys the party will depend on who is there.}
\]
In retrospect, it is easy to see why this shift of attention to embedded questions was so fruitful since in the case of direct questions it is much harder to distinguish the contribution of the semantics of the question from the contribution of the pragmatics of the speech act of asking (or any of the other speech acts that can employ question forms).

There are still many competing analyses for the semantics of questions, and the more satisfactory ones are too complex to sketch very briefly. Let me just mention that one recent magnum opus on the topic, Groenendijk and Stokhof (1984), concludes that a satisfactory analysis of the semantics of questions must include a kind of double nesting of intensionality, or double layering of appeal to possible worlds. Expressed very informally, one might say that a question such as "Who Mary loves" is interpreted as a function which to each possible world assigns the proposition (set of possible worlds) that expresses the true and complete answer to that question in that world.

The arguments for the necessity of such double-layering of possible worlds are complex and technical; but I think at the very least all would agree that the tools provided by possible-worlds theory have yielded enormous advances in the level of argumentation and the adequacy and explicitness of analyses of the semantics of questions. I think the developments in the analyses of questions have also been a good example of the value of simultaneously working to develop explicit theories of syntax, semantics, and pragmatics, and their interaction. The compositionality requirement, constraining the relation of syntax and semantics, forced careful attention to the semantics of embedded questions, which occur not only with verbs of asking but with verbs of knowing, doubting, wondering, and such miscellaneous constructions as the depends on construction. This in turn enabled researchers for the first time to begin to clearly distinguish between the semantic and pragmatic components that contribute to the interpretation of a direct question, which in turn has helped to sharpen our understanding of the relation of semantics to pragmatics more generally.

As a last and quite different example, I will just mention the contribution of possible worlds to the development of pragmatics more generally, and in particular to the analysis of presupposition and of the role of context and context change in the interpretation of discourse. In fact I think it is fair to say that within the formal semantic tradition, possible worlds have been partly responsible for the shift of attention from the interpretation of sentences in isolation to the interpretation of discourse in context.

This shift goes back to Stalnaker's work (Stalnaker 1974, 1976, 1978) and he has played a leading role in its development, with recent work by Kamp and especially by Heim (1982, 1983a, 1983b) giving it an added thrust in the linguistic community. I have already mentioned Stalnaker's notions of common ground and context set and alluded to Heim's extensions of them. The connection to the notion of presupposition is approximately as follows. From the time of Frege, and reinforced by Strawson, presuppositions of a sentence $S$ were characterized as propositions whose truth was in some sense implied both by the truth of $S$ and by the truth of not-$S$. So, for example, both (13) and (14) seem to imply the truth of (15).

(13) John's children are asleep.
(14) John's children are not asleep.
(15) John has children.

But it is difficult to incorporate such a notion of presupposition in the semantics without invoking some kind of a 3-valued logic, and even with 3-valued logic, repeated attempts to work out a satisfactory theory of the computation of the presuppositions of a complex sentence from the presuppositions and semantic content of its parts had all been unsatisfactory in one way or another.

Stalnaker (1974) had early on proposed that the phenomenon of presupposition should be given a basically pragmatic analysis, in terms of requirements placed on the common ground in which an assertion is made. So for example, one can analyze (13) and (14) as being designed for use in discriminating among possible worlds in all of which (15) is true, and not felicitously usable in contexts in which (15) cannot be taken for granted, either by already being common knowledge or by the listeners' willingness to treat it as if it were already common knowledge.

This proposal seemed plausible enough in the case of the presuppositions of simple sentences, but it did not by itself satisfy the linguists' concern with the problem of computing the presuppositions of complex sentences; that problem seemed to require either a semantic solution or a much tighter integration, even sentence-internal, of pragmatic and semantic operations, which seemed contrary to common assumptions about the nature of pragmatics. The works of Stalnaker, Kaplan, Kamp, Krasner and Heim mentioned earlier, and potentially some of the ideas of Barwise and Perry for relating discourse situations, resource situations, and described situations, has gradually succeeded in convincing many scholars that semantics and pragmatics indeed have to be more closely integrated, or the boundaries between them redrawn, so that the very fabric of compositional semantic interpretation is set up not just to arrive at a set of possible worlds as the interpretation of a sentence, but as a complex function which (a) imposes constraints on the context on which it is interpreted, (b) contributes a proposition in the old sense if those constraints are met, and (c) effects certain changes in the context in the process. (This kind of approach is most clearly articulated in Heim (1982) and Heim (1983b).) There is more to context on this view than just a set of possible worlds determined by the common ground; but the possible worlds are indeed a crucial part of the context.

In closing this section let me mention one possible challenge to the kinds of claims I have been making, which have by and large been claims about the beneficial effects of possible worlds theorizing on the development of linguistic semantics (and pragmatics). Aside from the last remarks about pragmatics, one might charge that all the advances I have discussed could be described in terms of...
analyses expressed in Montague’s intensional logic or some variant of it, and the fact that that logic is interpreted in terms of possible worlds is inessential. Charges of this general form are sometimes leveled by representationalists who are willing to entertain something like intensional logic as a candidate language of semantic interpretation, but who remain unconvinced that the model theory is doing any real work here.

This is a complex kind of charge to try to answer, though I think it is important to try.

First of all, I believe the claim is only correct, if at all, for analyses carried out within a given paradigm such as Montague’s intensional logic; whenever formal semanticists or philosophers argue for a need for a modification or extension of the framework, as has in fact been the case for many of the examples I have discussed here, the arguments have not been just for a new notation but for an explicit model-theoretic semantics as well—or sometimes for the latter directly without any new notation. Since Montague’s intensional logic has no complete axiomatization, being explicit about the model theory is in many cases the only way to be explicit about the semantics at all.

Secondly, if one looks closely at discussions of the semantics of propositional attitude sentences, one finds that notation has very little to do with the debates about interpretation. It is ironic that what seems to be the hardest problem in semantics is manifested by a syntactic construction that seems utterly straightforward and probably universal: a propositional attitude verb followed by an embedded sentence, with nothing tricky about the syntax of the embedding. In Montague’s intensional logic, the semantic notation is equally straightforward: just put an intension operator on the sentence. In thinking about proposals for alternative treatments that might be more satisfactory, concern with notation plays a very subsidiary role compared to concern with what entailments one thinks should follow and where they come from—clearly semantic concerns in a model-theoretic sense. The same kinds of remarks could be made about the hard issues in the semantics of counterfactuals.

So even if possible worlds turn out to be dispensable in the end, and even if appeal to possible worlds raises as many questions as it answers, I believe there is no denying that by the linguists’ primary standard of fruitfulness in generating explanatory accounts of linguistic phenomena, the use of possible worlds in semantic theories has had a far-reaching positive impact, and that the role of possible worlds in that influence has been essential.

IV. Foundational Issues

In this final section, I want to mention some of the important foundational issues I see facing the further development of possible-world semantics and pragmatics and the elucidation of the conception of possible worlds that can best underlie it.

(1) Can possible worlds semantics be reconciled with conceptualism? If one reads just Montague or the logicians in whose tradition Montague was working, one sees semantics treated on its formal side as a branch of pure mathematics and on its empirical side as dealing with questions about the relation of language to the world, e.g., questions about the actual truth conditions of various sentences. None of this looks very inviting to a linguist who takes the study of semantics to be primarily a study of the nature of language users’ semantic competence.

Reading Stalnaker (1984), however, one sees more clearly that possible-worlds semantics can just as readily be viewed as a theory of the content of psychological states associated with the asserting or accepting of various sentences, though not directly as a theory of the mechanisms employed by the language users themselves. That is, it is in principle perfectly compatible with conceptualism if we take the possible worlds semantics as providing an “external” characterization of the relevant concepts and not an “internal” one.

As Stalnaker has put it elsewhere (ms. 1985b), possible worlds semantics as he understands it is a framework for raising issues in philosophy, psychology, and linguistics, not a particular set of philosophical beliefs.

In earlier writings I had raised some worries about the possible psychological reality of possible worlds which in retrospect I think arose partly from misguided concern with trying to fit possible-worlds theory “inside the speaker’s head”: but we don’t try to fit the theory of gravitation “inside a falling apple”. Nevertheless, linguists do care about mechanisms, and if possible-worlds semantics is a reasonable theory about the language user’s semantic competence, there still have to exist some internal mechanisms partly by virtue of which the external theory is correct.

Just to give one sample of the difference between external theory and internal mechanism as I now see the distinction, consider the question of how many possible worlds there might be. Even on a non-absolutist view where we don’t take that question to have a single determinate answer, on many applications there will be at least non-denumerably many possible worlds. But that does not mean that a theory of internal mechanisms has to provide a way to represent each of non-denumerably many possible worlds: for our discriminations of possible worlds from each other are always at the level of discriminating certain sets of possible worlds from other sets of possible worlds. The sentences we use to express various propositions might themselves be candidates for representations of sets of possible worlds! Though if it turned out that those were the only or the canonical representations of sets of possible worlds, one might well be inclined to wonder whether we weren’t coming back full circle to a representationalist view after all. But if it is correct that sets of possible worlds serve to interpret the content of other psychological states such as perception, desire, etc., then it seems likely that internal mechanisms of representation, like the external theory, will employ some kind of medium less specifically linguistic than sentences, whether in English or in a language of thought. In any case, an internal mechanism might be classifying possible worlds into sets on just some
finite number of parameters at the same time that a correct external description of the corresponding belief, assertion or whatever involved quantification over an infinite set of possible worlds.

(2) A non-absolute picture seems to fit linguistic semantics better than an absolutist one, where by absolutist I mean the position that there is one single maximal set (or class, if it's too big to be a set) of possible worlds. If a philosopher could find arguments that in the best metaphysical theory there is indeed a maximal set, I suspect that would for the linguist be further confirmation that his enterprise is not metaphysics, and I would doubt that such a maximal set would ever figure in a natural language semantics.

As various people have noted, possible worlds are really not so different in this respect from entities: every model-theoretic semantic theory I'm familiar with takes entities to be among the primitives - but puzzles about the identity conditions of individuals and about whether there is a maximal set of all of them are just as problematic, and it is just as questionable whether semantic theory has to depend on settling such questions. As emphasized in section III, it is the structure provided by the possible worlds theory that does the work, not the choice of particular possible worlds, if the latter makes sense at all. 7

(3) "Conceivable worlds" are not the same as "possible worlds"; does concern with a conceptualist foundation for natural language semantics require that one work with the former instead of the latter? No, I don't think so. Some "conceivable" states of affairs may in fact be impossible, and some possible worlds may be beyond our powers of conception. I think that even the simple fact that we can conceive of there being possibilities we can't conceive of shows that such questions have to be approached with caution. The questions that have to be addressed in order to sort out these issues include both a further elucidation of the distinction between the external and the internal perspective on the human agent that I mentioned above, and questions about the nature of reflecting on one's own knowledge and beliefs, questions that are known to lead to deep philosophical and logical puzzles around which the scent of paradox is often hanging in the air.

Furthermore, I'm not sure that the adjective "conceivable" should even be applied to individual worlds in the way that the adjective "possible" is. Given what we said above, discriminating single individual possible worlds from all other possible worlds may be entirely beyond our powers of conception, which may be limited to discriminating among sets of possible worlds. So the notions may be even more incomparable than one would have first supposed. In any case, I am convinced that it is possible worlds and not conceivable worlds that we should posit for most of natural language semantics, though I recognize that the relation between these notions needs much more investigation. 8

(4) Current research on property theory as an alternative to set theory, such as reported in Bealer (1982), Chierchia (1982), Turner (1987), Chierchia and Turner (ms. 1985) and others, is bound to substantially reshape our thinking about possible worlds. Recall that intensions are treated in possible-worlds semantics as functions from possible worlds to extensions. But the logical equivalence problem suggests that this may not be a sufficiently intentional notion of intension. Part of the problem may lie in the fact that these functions from possible worlds to extensions are still functions in an extensional sense - functions completely representable as sets of ordered pairs. What if we had a more thoroughly intentional property theory at the foundations of the whole theory, rather than an extensional set theory? If propositions were analyzed as properties of possible worlds rather than sets of possible worlds, the logical equivalence problem might not arise, at least not in the same form. On the other hand, if we had properties as primitive, and propositions as well (treating these as zero place properties), it is also less clear that we would need possible worlds as crucially as we do when we analyze propositions as sets of possible worlds and properties as functions from possible worlds to the sets of individuals that exemplify them.

(5) The propositional attitude problem, as I indicated above, is still to my mind the hardest problem in the foundations of semantics. Saulkner (1984, ms. 1985a) has done quite a remarkable job of defending the classical possible-worlds analysis of propositional attitude attributions against criticisms, but many linguists and philosophers remain unconvinced of the success of his attempts to explain away the logical equivalence problem. Yet repeated attempts to invoke a more "finely-grained" analysis of propositions have met considerable difficulties as well. Nevertheless, it does seem that progress is being made, so the problem should not be dismissed as utterly hopeless.

One particular point to watch out for: if some other analysis of propositions, not based on possible worlds, turns out to do the best job for the semantics of propositional attitude attributions, the possibility or utility of possible worlds for other purposes will undoubtedly need to be subjected to very close scrutiny.

(6) One alternative to possible-worlds semantics that I haven't discussed so far is algebraic semantics. I don't feel competent to discuss it with any authority, but as I understand the enterprise it might be viewed as a strategy of abstracting out the relevant structure in what might have started as a model-theoretic semantics, abstracting away from all properties of the original models which don't directly affect their algebraic structure. For instance, I suppose looking at propositional logic as a Boolean algebra is an example - once one sees the

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7 This emphasis on the structures provided by the theory rather than the substance of its parts can be seen as a major motivation for the further step to "algebraic semantics" mentioned below as item (6).

8 But see the arguments for the contrary position in Thomas Kuhn's paper in this volume. We both agree that trying to resolve this issue takes us well beyond the scope of both our papers.
Boolean structure, it doesn’t matter directly what the elements of the algebra are: all that matters are their algebraic connections to each other.

Such concern with structure is quite congenial to the linguistic enterprise in general, so if possible-worlds semantics were to take such a turn I would imagine linguists would be perfectly happy to keep the structure of possible-worlds semantics while ignoring the possible worlds themselves for the most part. However, I would expect that as in any process of scientific modelling, when it came to making judgments about competing theories offering different abstractions of the relevant structure of the possible worlds, part of the basis for comparative judgments would be how faithful the abstractions were to the things being modelled, i.e. the possible worlds. But on the other hand, if the possible worlds themselves are, for semantics, just a useful theoretical poset (from the linguist’s perspective at least), then a linguist’s judgment would probably be based more on the predictions the theory made about judgments about various sentences than on what it claimed about the possible worlds. As far as the working linguist is concerned, possible worlds could conceivably end up going the way of caloric or phlogiston if the theories they figure in were supplanted. One might then be able to divide the “real linguists” from the “real philosophers” among those engaged in the currently very interdisciplinary work on formal semantics – the linguists would be those who could let the possible worlds go without a twinge, and without thereby denying their existence any more than current linguistic work in possible-worlds semantics depends on the metaphysical reality of possible worlds.

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