Meaning Exchange Games

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Research supported by ERC Grant 269427

August 2015

What we saw last time

- Gricean communication
- Grice cooperativity as formalized in Perrault (1987) is provably equivalent to aligned preferences.
- Problems with non-cooperative settings
- troublesome backwards induction argument,
- completely avoided by moving to infinitary game framework.

This lecture:

- more motivations for our approach
- varieties of infinitary games
- BM games and ME games
-

What are we interested in?

- how to characterize message exchanges, winning and losing a conversation,
- in a strategic context where user's preferences may be incompatible.
- the general shape of a conversation as characterized by players' goals
- aren't some conversational goals more difficult to achieve than others?

Example 1

Interview

Suppose a candidate, Candidate A, has a joint interview with another competing candidate, Candidate B, for an academic position. Suppose Candidate A has proved an important theorem and she knows that during the interview if she can mention this, she will have "won" the interview by getting the job over the smarter candidate B, as long as she can mention this fact, no matter at what point of the meeting she says so. This is her winning condition.

Example 2

From Solan & Tiersma *The language of Crime*

(2a) Do you have an account in that bank, Mr. Bronston?.

(2b) Bronston: No, sir.

(2c) P: Have you ever?

(2d) B: The company had an account there for about six months, in Zurich.

Example 2

From Chris Potts p.c.

(2'a) Justin: Have you been seeing Valentino?

(2'b) Janet: He has mono.

Example 3

Clinton's campaign

Recall President Clinton's adage "it's the economy stupid." What Clinton meant is that he should keep the conversation focussed on questions concerning the economy in the extended debate between his Democratic team and the opposing Republican one during the 1992 Presidential campaign. As long as Clinton was able to bring the debate repeatedly back to a discussion of the economy, he achieved his winning condition.

Example 4

Quayle-Bentsen Vice presidential debate

Quayle was repeatedly questioned about his experience and his qualifications to be President. Quayle's attempted to compare his experience to the young John Kennedy's to answer these questions. However, Bentsen made a discourse move that Quayle didn't anticipate.

4a . Quayle: ... the question you're asking is, "What kind of qualifications does Dan Quayle have to be president," [...] I have as much experience in the Congress as Jack Kennedy did when he sought the presidency.

4b Bensten: Senator, I served with Jack Kennedy. I knew Jack Kennedy. Jack Kennedy was a friend of mine. Senator, you're no Jack Kennedy.

4c Quayle: That was unfair sir, unfair.

4d Bentsen: You brought up Jack Kennedy. I didn't.

Example 5

Feynmann

Allegedly, the physicist and Nobel laureate Richard Feynmann decided the topics of his next lecture in advance and prepared for it for over 8 hours. However, when he entered the class he would start off with: *So what shall we discuss today?* But he would always have a strategy to steer the conversation to the topics he had prepared for, whatever his students, who always wanted to stump him (and so had opposing interests to Feynmann's), would answer.

bargaining

- with the goal of eventually getting a stable agreement

Example 6

Voire dire examination from Malone (2010)

The plaintiff lawyer (LP) repeatedly comes back to questions for a defendant.

6a LP: And also, he put an electrical signal on that nerve, and it was dead. It didn't do anything down in the hand, it didn't make the hand twitch?

6b D: Correct.

6c LP: And we know in addition to that, that Dr. Tzeng tore apart this medial antebrachial cutaneous nerve?

6d D: Correct.

6e LD: Objection.

6f D: Correct. There was a division of that nerve. I'm not sure I would say "tore apart" would be the word that I would use.

6g LP: Oh there you go. You're getting a hint from your lawyer over there, and so you want to retract what you're saying?

Basic principles

1. People have conversations for purposes. Their conversations are successful when they achieve those objectives. Crucially, some of these objectives involve commitments by other conversational participants.
2. Some conversations are win-lose or at least zero sum games, where one person's gain is another person's loss. Nevertheless, even in such games, people do converse and their contributions have a set meaning.
3. In principle, conversational players have no limits on the length of their intervention, though they are finite.
4. Players can in principle "say anything" during their conversational turn, though what they say may very well affect whether their conversation is successful or not.
5. While conversations are finite, they may have no designated "last turns;" conversational agents cannot in general foresee who will "have the last word."

Models

- So model conversations as infinite games of a particular type.
- players win strategic conversations by pursuing objectives over an indefinite number of turns.
- winning objective is often a property of the string of moves themselves. (candidate replied well to all the questions)

Varieties of infinite games

- 0 sum games, special case of richer non 0 games.
- well understood connections to topology, logic, and verification of reactive systems in CS.
- useful in many areas, e.g. in showing fundamental properties of learning theories
- determinacy properties.
- Banach Mazur games (Scottish book 1930's), (Gale Stewart games (Gale Stewart 1953), Wadge games, Blackwell games

Gale Stewart games

- (V^{ω}, Win) with 2 players, 0 and 1.
- over a non-empty set V
- the players take turns in playing 'single' elements from X. Win is a subset of X^{ω}
- Player 0 wins a play if and only if $\rho \in Win$.

BM games

- (V^ω, Win)
- 2 players, at turn i, each picks a finite sequence v_i of such moves, with $v_i \in V^*$. V
- Players alternate indefinitely, building strings in V^ω
- A BM game contains a winning condition for player 0 Win. 1's winning condition is understood to be $V^\omega \setminus Win$.

Winning strategies

- A player's strategy is a function from the set of finite plays in X^ω to a finite sequence of discourse moves by her.
- a player 0 has a winning strategy in a game G iff no matter what moves 1 makes, 0 can move such that the resulting infinite play is in *Win*.
- 1 has a winning strategy in G iff 1 can keep the sequence in the complement of *Win* no matter what 0 does.

Determinacy

- A game is *determined* iff one of the players has a winning strategy.
- Chess is determined (Zermelo), but it is not known whether Black or White has the winning strategy.
- If G is a GS game where *Win* is a Borel subset of X^ω, G is determined [?].
- If G is a BM game where *Win* is a Borel subset of X^ω, G is determined [?].

More on BM games

- Define the flattening *flat* of a play $p = (x_k)_{k \in \mathbb{N}}$ as the infinite sequence eventually designed by the two players: $\mathit{flat}(p) = x_0 \cdot x_1 \cdot x_2 \ldots \in X^\omega$.
- Player 0 wins the game if $\mathit{flat}(p) \in \mathit{Win}$. Player 1 wins otherwise.

Proposition

Let $BM(\langle V^\omega, \mathit{Win} \rangle)$ be a BM-Game. Then, for any play p and every play $p' \in \mathit{flat}^{-1}(\mathit{flat}(p))$, player i wins in p iff player i wins in p'.

Linguistic BM games

- alphabet X: basic discourse moves involving discourse relations between units.
- X^ω are all the possible conversations using this alphabet.
- A conversational BM game: (X^ω, Win).
- in standard signaling games, messages are not analyzed, and meanings can be anything.
- Farrell's (1993) approach investigating messages with a fixed meaning outside of equilibrium seems more promising. Focus on public commitment as opposed to credible information.
- we'll see how meanings are determined in a bit, via the Jury.

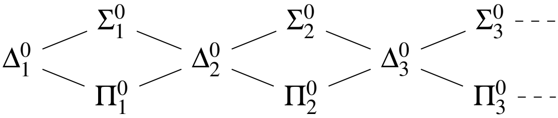
Topology

- Continuations of example 2:
  2d P: I'm interested in whether you, Mr. Bronston, ever had a bank account, (f) not about your company (g) Please answer the question.
  2e P: Thank you, Mr. Bronston I now would like to move to the question of your involvement in the Victory offshore trading company.
- The set $O(2*2e) \cup O(2*2e)$ is the set of all continuations of the two possible sequences, where $O(a) = a \cdot V^{\omega}$ is a basic open set.
- $\overline{O(1*4)} \cap \overline{O(1*5)} = \overline{O(1*4) \cup O(1*5)}$ is a closed set, the ways the conversation could have deviated either from $1*4$ or from $1*5$.
- generates a topology over the strings in V^{ω}

A few definitions

- . A set is dense if it intersects every open set.
- A set is nowhere dense if its complement contains a dense open set.
- A set is meager if it is a countable union of nowhere dense sets.
- A set is co-meager if is a countable intersection of dense sets.
- $\overline{O(1*4)} \cap \overline{O(1*5)} = \overline{O(1*4) \cup O(1*5)}$L the ways the conversation could have deviated either from $1*4$ or from $1*5$.

Borel Hierarchy



Borel complexity and FO definability (V. Diekert and P. Gastin 2008)

Definition

a language $L \subseteq V^{\omega}$ is * free regular iff $\varepsilon \in L$ if $a, b \in L$ then $ab \in L$ and $a|b \in L$

Definition

A finite monoid M is aperiodic if for all $x \in M$ there is some $n \in N$ such that $x^n = x^{n+1}$

Definition

A language L is aperiodic if it is recognized by some morphism to a finite and aperiodic monoid; i.e., $L = X^*.Y^{\omega}$ for $X, Y \subseteq V$

Two theorems from Theoretical CS

Theorem

*A language L is FO $<$ definable $\equiv$ L is LTL definable (Kamp) $\equiv$ L is a *-free regular language $\equiv$ L is a-periodic (McNaughton and Papert FO $\equiv$ * free; * free $\equiv$ a-periodic (Schutzenberger, Perrin).*

Corollary

FO definable languages $\subsetneq \Sigma^0_2 \cup \Pi^0_2$ Borel definable sublanguages on V^ω; in particular, FO picks out a strict subset of the $\Sigma^0_2 \cup \Pi^0_2$ Borel conditions.

Strategies again

- A player's strategy is a function from the set of finite plays in X^ω to a finite sequence of discourse moves by her.
- A winning strategy for player i is a function that assures i of attaining her winning condition no matter what her opponent does.

Theorem (Banach-Mazur)

Given a BM game $BM(X^\omega, Win)$, (i) Player 1 has a winning strategy if and only if Win is a meager set; (ii) Player 0 has a winning strategy if and only if, there exists a finite string x such that $O(x) - Win$ is meager (that is, Win is co-meager in some basic open set).

Idea of Proof (Graedel 2009)

- Look at the case of player 1: For any strategy g of Player 1 in a Banach-Mazur game on a graph (G, v), the set Plays(g) of all plays that are consistent with g is a countable intersection of dense open sets.
- $\text{Plays}(g) = \bigcup_{n\in\omega} \text{Plays}_n(g)$, where $\text{Plays}_n(g)$ is the set of all plays compatible with $g$determined by 1's n first moves.
- Each Plays_n is open, but also dense since 0 can play any finite sequence x in her first move.
- Since $\text{Plays}_n(g)$ is dense, then for any finite prolongation of x, y, $\mathcal{O}(xy) \subseteq \text{Plays}_n$.
- g is a winning strategy iff $Win \cap \text{Plays}_n(g) = 0$ for each n: i.e. $Win = \bigcup_{n\in\omega} \overline{\text{Plays}_n(g))}$
- So 1 has a winning strategy iff Win is a countable union of nowhere dense sets, i.e. meager.

Limitations of BM games

- Consider example 2. There's an important difference between Bronston's response to a question by the prosecutor and the prosecutor's offering that information himself.
- that difference can't be captured in BM games.
- Discourse moves contain more information than the sentence itself. Bronston's committing to a negative answer to (2a) provides more information than just the string *no sir*.

Message Exchange (ME) Games

- An ME game involves a vocabulary of discourse moves V that yields two disjoint sets $V_0 =_{def} V \times \{0\}$ and $V_1 =_{def} V \times \{1\}$, standing for the respective vocabularies of 0 and 1.
- The game is played with 0 and 1 alternatively choosing finite sequences V_0 and V_1 A turn by i is an element of the set $(V_i)^*$.
- The plays of the ME game over V form ω sequences in $(V_0 \cup V_1)^\omega$.

ME games defined

Definition

A 0 sum ME game is a pair $((V_0 \cup V_1)^\omega, Win)$, with $Win \subseteq (V_0 \cup V_1)^\omega$.

Definition

A non 0 sum ME game is a pair $((V_0 \cup V_1)^\omega, Win_0, Win_1)$, with $Win_0, Win_1 \subseteq (V_0 \cup V_1)^\omega$.

Remark

Though we have assumed 0 and 1 share a common set of discourse move types, we can lift this assumption allowing $V_0 \subset V_1$ or $V_1 \subset V_0$. See example 4.

Rationality

Proposition

(i) Any winning strategy for player i in a zero-sum ME game paired with any strategy of $j \neq i$ defines a Nash equilibrium; (ii) Any pair of winning strategies for a non 0-sum ME game defines a Nash equilibrium.

Moves that depend on who makes them

- Let π denote the natural projection of $V_0 \cup V_1$ onto V ($\pi(\langle v, i \rangle) = v$).
- Define π_ω as the extension of π into a projection of sequences in $(V_0 \cup V_1)^\omega$ onto V^ω: $\pi_\omega((v_k)_{k \in \mathbb{N}}) = (\pi(v_k))_{k \in \mathbb{N}}$ (where all the v_k belong to $V_0 \cup V_1$).

Decomposition sensitivity

Definition (Decomposition sensitivity)

$Win \subseteq (V_0 \cup V_1)^\omega$ is *decomposition sensitive* iff
$\exists W \subseteq \pi_\omega(Win) \neg(\pi_\omega^{-1}(W) \subseteq Win)$
$Win \subseteq (V_0 \cup V_1)^\omega$ is *decomposition invariant* iff $\exists W \subseteq V^\omega \; Win = \pi_\omega^{-1}(W)$.

Proposition

Given an ME game G with a decomposition invariant Win_G, 0 will have a winning strategy in G iff she has a winning strategy in the BM game $BM(G) = (V^\omega, \pi_\omega(Win_G))$.

Winning conditions

- let's explore now more closely the character of winning conditions.
- in particular where does/can *Win* show up in the Borel Hierarchy?
- are all winning conditions FO, LTL definable?

Winning conditions, Σ_1^0

- Σ_1 winning conditions can be characterized by unions of basic open sets. Equivalent to *reachability* conditions.
- Suppose R is a subset of X. Then $Reach(R) = \{x \in X^\omega \mid R \subseteq occ(x)\}$ is the set of all strings which contain at least one element of R.
- Prosecutor in the Bronston e.g. is pursuing a Σ_1 objective getting an answer to his question.

Differences between BM and ME games

- Prosecutor's Σ_1 objective is co meager in a basic open set (itself).
- But his winning objective is turn involving (getting B to answer the question).
- Prosecutor has no winning strategy. (B always refuses to answer).
- BM theorem is false for turn involving winning conditions.

Winning strategies in ME games

Because of the turn structure we lose the tight connection in BM games between topology and the existence of winning strategies.
Define recursively:

- $\text{1-play}_0 = \{p.V^\omega | p \in V_0^*\}$
- $\text{1-play}_{n+1} = \{\text{1-play}_n.\sigma^0_{n+1}.\sigma^1_{n+1}.V^\omega | \sigma^0_{n+1} \in V_0^* \wedge \sigma^1_{n+1} \in V_1^* \wedge$ $\overline{Win}$ is dense in $\{O(\text{1-play}_n.\sigma^0_{n+1}.\sigma^1_{n+1}.\sigma) | \sigma \in V_0^*\}\}$

Proposition (General nec & suff condition)

1 has a winning strategy in an ME game iff $\bigcap \textit{1-play}_{n\in\omega} \neq 0$. *0 has a winning strategy iff* $\bigcap \textit{1-play}_{n\in\omega} = 0$.

- 1 has a winning strategy no matter what 0 says on the next turn.
- Proposition above collapses to BM theorem for decomposition insensitive *Win*.

Π_1 or Safety conditions in conversation

- Π_1 conditions are complements of Σ_1 conditions. Equivalent to *safety*.
- Suppose S is a subset of X (the 'safe' set). Then $Safe(S) = \{x \in X^\omega \mid occ(x) = S\}$ is the set of all strings which contains elements from S alone.
- That is, the strings remain in the safe set and do not move out of it.
- Bronston is pursuing a Π_1 objective. He wants to keep from giving answers to the questions posed by Prosecutor. Even though his objective is meager, he has a winning strategy (don't answer).

Co Büchi Winning conditions, Σ_2^0

- Σ_2 conditions are unions of of Π_1 sets. Co-Buchi conditions.
- In terms of temporal logic, such winning conditions are expressed as $\Diamond\Box\phi$—eventually you will always be in a state characterized by ϕ.
- Feynman example, voire dire example.
- information seeking dialogues, where eventually 0 establishes a truth that is no longer contested are examples of conversational games with Σ_2 winning conditions.
- bargaining dialogues are similar; they aim at a stable agreement. Turn-involving conditions.
- Puzzle: there are no winning strategies for Σ_2 turn involving conditions with eventual agreement (always disagree). So why do bargains happen in Win Lose games? (epistemic limitations).

Buchi conditions, Π_2^0

- Π_2 conditions are complements of Co-Buchi conditions.
- In terms of temporal logic, we could think of a fragment of winning conditions in this class in terms $\Box\Diamond\phi$—you are always able to visit a state (or property of strings) characterized by ϕ.
- Clinton's "it's the economy, stupid". (keep on coming back to the economy).
- Consider Example 4. Our excerpted example was a turning point in the Vice-Presidential debate. Quayle's goal as Player 0 was to continually revisit the theme that despite his youth he had the talent and experience of a good Vice-Presidential and Presidential candidate. This is a Π_2^0 winning condition.

Muller conditions

- Suppose we are given a set $\mathscr{F}$ of subsets of X (the Muller sets). Then $Muller(\mathscr{F}) = \{x \in X^\omega \mid inf(x) \in \mathscr{F}\}$ is the set of all strings which eventually (after a finite point) revisit infinitely often one of the Muller sets
- A Muller winning condition is a boolean combination of Büchi and Co-Büchi conditions. Let $\mathscr{F} = \{F_1, F_2, \ldots, F_n\}$ be the set of Muller sets where each F_i is a subset of X. Then
- $$\phi_{Muller(\mathscr{F})} = (\phi_{co\text{-}Büchi(F_1)} \vee \ldots \vee \phi_{co\text{-}Büchi(F_n)}) \wedge (\phi_{co\text{-}Büchi(F_1)} \Rightarrow \bigwedge_{x \in F_1} \phi_{Büchi(\{x\})}) \wedge \ldots \wedge (\phi_{co\text{-}Büchi(F_n)} \Rightarrow \bigwedge_{x \in F_n} \phi_{Büchi(\{x\})})$$

Applications to conversation

- Revisiting the same information states infinitely often implies these are not information seeking games
- "You don't love me." "I'm fat." "I'm old and ugly". "I'm stupid."
- *a question ain't really a question if you know the answer too.* (John Pryne *Far from me*)
- Such conversational behaviors call out for an explanation. (a hidden message that the speaker is implicitly conveying).

Beyond Muller

- A strategy is positional if an automaton with a single state encoding the last element of the last move can compute it.
- All winning conditions in BM games of complexity up to an including Muller have positional winning strategies (Graedel 2008).
- The situation is different in ME games—e.g. consider the condition of getting an answer to a question (have to remember whether it was answered or not in perhaps a very long response by opponent).

More winning conditions

Consistency

Our ME games allow a player to say anything on her turn. This misses linguistic constraints intrinsic to a good or winning conversational strategy.

Definition

A play ρ of an ME game in V^{ω}, with V a finite set of negation closed litterals, is consistent for player i iff $\rho \upharpoonright V_i$ does not both contain p and $\neg p$ for any letter $p \in V$.

remarks on the language of discourse moves

These can help us formulate our constraints more precisely.

- discourse moves are relational
- dialogue structure is a graph (V,E) with V the set of discourse units, and E the set of edges in $V \times V$ representing discourse relations.
- We write $attack(\pi, \pi')$ when there is an edge $e(\pi, \pi')$ in the discourse graph that is labelled 'attack', and this happens discourse move π' attacks a commitment that depends on discourse move π.
- Similarly, we write $response(\pi, \pi')$ when there is an edge $e(\pi, \pi')$ in the the discourse graph that signals that π' is a response to π (which is typically an attack on π).

Exploiting the turn structure of ME games

- Let $\mathscr{T}$ be the set of turns (each turn is a pair of moves by 0 followed by moves of 1) in an ME game
- let $\mathsf{proj}_i : \mathscr{T} \rightarrow \wp(DU)$ be the projection from a turn to the set of DU's of the contribution by i therein.

Rhetorical cooperativity

- **Coherence** A contribution by Player $i \in \{0,1\}$ is coherent on turn τ if for all $\pi \in \mathsf{proj}_i(\tau)$ there exists $\pi' \in (\mathsf{proj}_i(\tau') \cup \mathsf{proj}_{1-i}(\tau'))$, where τ' is τ or some previous turn, such that there exists an edge $e \in E$ such that we have either $(e(\pi', \pi)$ or $e(\pi, \pi'))$. That is, the contribution in τ links to a previous contribution by either player.
- **Responsiveness** Player $i \in \{0,1\}$ is responsive on turn τ if there exists $\pi \in \mathsf{proj}_i(\tau)$ such that there exists $\pi' \in (\mathsf{proj}_{1-i}(\tau'))$ where τ' is the previous turn such that for some $e \in E$ we have $e(\pi', \pi)$.

Replying to attacks

NEC NEC holds for Player $i \in \{0,1\}$ on turn τ iff for all $\tau' < \tau$ for all $\pi_0 \in \rho_i(\tau')$ for all $\pi' \in \cup\rho_{1-i}(\tau')$ where $\text{Attack}(\pi_0, \pi')$ and there is no $\pi'' \in \rho_i(\tau')$ such that $\text{Response}(\pi', \pi'')$, there exists $\pi \in \rho_i(\tau)$ such that $\text{Response}(\pi', \pi)$

A refinement: CNEC

CNEC holds for Player $i \in \{0,1\}$ on turn τ of a ρ if there are fewer attacks on 0 with no response in $\mathsf{proj}_{1-i}(\tau'))$ for $\tau' \leq \tau$ than for $1-i$. **CNEC** holds for Player $i \in \{0,1\}$ over a play ρ if in the limit there are more turns of ρ where CNEC holds for i than there are turns of ρ where CNEC holds for $1-i$ [we shall make this notion formal later, after we introduce the model of the jury].

The Jury

- who enforces consistency, etc.? After all, our players can say anything.
- And they could also lie about their attaining their winning conditions!
- the Jury (cf. Rubinstein's work on persuasion games)
- an abstract yardstick for gauging success in conversation.
- But it could be: a courtroom jury, one of the players, the general electorate, or even one or both of the players.
- the Jury enforces minimum standards of consistency, rhetorical cooperativity, and also reputation effects.

the Jury's rating

The Jury assigns a rating, $\|\tau_k\| \in \mathbb{R}$, to the contribution in turn τ_k with the following constraints:

- if the player 0 in τ_k fails to respect coherence in τ_k then

$$\mathsf{coh}_0(\tau_k) = \begin{cases} -1 & \text{if } k \bmod 2 = 0 \\ 1 & \text{otherwise} \end{cases}$$

- if the player of τ_k is not responsive in τ_k, then

$$\mathsf{res}_0(\tau_k) = \begin{cases} -1 & \text{if } k \bmod 2 = 0 \\ 1 & \text{otherwise} \end{cases}$$

- If 0 is inconsistent by turn τ_k of ρ and 1 is not, then $\mathsf{cons}_0(\tau_{k'}) = -1$ for all $k' \geq k$. Otherwise, $\mathsf{cons}_0(\tau_{k'}) = 0$
- In addition the jury also assigns a value $\mathsf{win}_0(\tau_k)$ to every turn τ_k as follows. Suppose ρ_{k-1} is the play so far and $\rho_k = \rho_{k-1}\tau_k$. Then $\mathsf{win}_0(\tau_k) = 1$ if $\mathscr{O}(\rho_k) \cap Win \neq \emptyset$. Otherwise $\mathsf{win}_0(\tau_k) = -1$. That is, $\mathsf{win}_0(\tau_k) = 1$ if τ_k advances 0 towards *Win*; $\mathsf{win}_0(\tau_k) = -1$ if τ_k takes 0 further away from *Win*.

the Jury's estimation of player types

- The Jury also maintains a probability distribution over types: BAD_0 and GOOD_0
- modeling the gain or loss of credibility that 0 has faced so far. At each turn we write this probability as P_k, and it is defined as follows:

the details

- $P_0(\text{GOOD}_0) = 1$ and $P_{k+1}(\text{GOOD}_0) = P_k(\text{GOOD}_0|\rho_k)$, where ρ_k is the initial sequence of k turns in the game.
- $P_k(\text{BAD}_0) = 1 - P_k(\text{GOOD}_0)$.
- if 1 successfully attacks 0 at turn k, then $P_k(\text{GOOD}_0) = P_{k-1}(\text{GOOD}_0|\rho_k) = c_k P_{k-1}(\text{GOOD}_0)$ where $0 \leq c_k < 1$ is a constant representing the severity of punishment per single move of a player i by the jury ($c_k = 2/3$ is a good example).
- Conversely, if 0 successfully attacks 1 at turn k (this includes a good response to an attack move by 1), then $P(\text{BAD}_0|\rho_k) = c_k P_{k-1}(\text{BAD}_0)$.

Scoring debates

These two ingredients contributes to a definition of the Jury's evaluation in the following way: $\|\tau_k\|$ of the k^{th} turn's benefits to 0 is given as:

$$\|\tau_k\| = \mathsf{coh}_0(\tau_k) + \mathsf{res}_0(\tau_k) + con_0(\tau_k) + P_k(\mathrm{GOOD}_0)(win_0(\tau_k))$$

And 0's score for a play ρ is given as

$$\|\rho\|_0^{\uparrow} = \liminf_{n \to \infty} \left[\frac{1}{n} \sum_{k=1, \tau_k \in \rho}^{n} \|\tau_k\| \right]$$

Jury condition

For a 0-sum game, 0 obeys the *Jury condition* for a sequence ρ only if $\|\rho\|_0^{\uparrow} > 0$ and 1 wins otherwise.
For non 0-sum ME games, we duplicate the clauses for 0 in the Jury Winning Condition to apply with the appropriate substitutions to 1. We then say as before that 0 wins iff $\|\rho\|_0^{\uparrow} > 0$ but that 1 wins iff $\|\rho\|_1^{\uparrow} > 0$. If neither of these obtain, then both players lose; if both obtain, they both win.

Complexity of Constraints

Proposition

$\mathsf{NEC}, \mathsf{COH}, \mathsf{RESP}$ *and* CONS *are* Π^0_2 *sets.*

The arguments for RESP ($\mathsf{NEC}, \mathsf{COH}$ and CONS are similar):
RESP can be written as follows:

$$\mathsf{RESP} = \bigcap_{n>0} \{\rho \in (V_0^+ \cup V_1^+)^\omega \mid \rho_n \textit{ is responsive in each turn}\}$$

where ρ_n is the length-n prefix of ρ. Now

$$\{\rho \in (V_0^+ \cup V_1^+)^\omega \mid \rho_n \textit{ is responsive for each turn}\}$$

is an open set. Moreover, it is not closed because otherwise its complement has to be open. However, the complement can be easily seen to be a countable intersection of open sets and hence cannot be open. Thus RESP, being the countable intersection of open sets, is Π^0_2.

Complexity continued

We now turn to the complexity of CNEC. The set $\mathsf{CNEC} \subset (V_0^+ \cup V_1^+)^\omega$ is defined as

$$\mathsf{CNEC} = \left\{ \rho \in (V_0^+ \cup V_1^+)^\omega \mid \liminf_{n\to\infty} \frac{\textit{good attacks by 0 in } \rho_n}{\textit{good attacks by 1 in } \rho_n} \geq 1 \right\}$$

That is, Player 0 has the upper hand over her opponent more often. Now by the definition of lim inf we have the following equalities

$$\mathsf{CNEC} = \bigcap_{N>0} \left\{ \rho \in (V_0^+ \cup V_1^+)^\omega \mid \exists m > 0,\ \forall n > m\ \frac{\textit{good attacks by 0 in } \rho_n}{\textit{good attacks by 1 in } \rho_n} > 1 \right.$$

$$= \bigcap_{N>0} \bigcup_{m>0} \bigcap_{n\geq m} \left\{ \rho \in (V_0^+ \cup V_1^+)^\omega \mid \frac{\textit{good attacks by 0 in } \rho_n}{\textit{good attacks by 1 in } \rho_n} > 1 - 1/N \right\}$$

Complexity of CNEC continued

Note that the set

$$\bigcap_{n \geq m} \left\{ \rho \in (V_0^+ \cup V_1^+)^\omega \mid \frac{\textit{good attacks by 0 in } \rho_n}{\textit{good attacks by 1 in } \rho_n} > 1 - 1/N \right\}$$

is closed. So, CNEC is a Π_3^0 set. Using ideas from Chatterjee (2007) we can show that CNEC cannot be expressed as a set of Borel complexity ≤ 2. That is, CNEC is Π_3^0 hard. We thus have

Proposition

CNEC *is a* Π_3^0 *complete set*

Final remarks

- Discourse coherence (responding to opponent's assertions and questions also requires memory makes winning a conversation more difficult.
- ME games build on BM games but introduce new and interesting complexities
- A rich universe of constraints and winning conditions
- we need to investigate meaning in such games
- and determinacy...