Iffiness∗

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1 An iffy thesis

One thing language is good for is imparting plain and simple information: \textit{there is an extra chair at our table} or \textit{we are all out of beer}. But — happily — we do not only exchange plain information about tables, chairs, and beer mugs. We also exchange conditional information thereof: \textit{if we are all out of beer, it is time for you to buy another round}. That is very useful indeed.

Conditional information is information about what might or must be, if such-and-such is or turns out to be the case. My target here has to do with how such conditional information manages to get expressed by sentences in natural language — sentences often dubbed “indicative conditionals”, not because anyone thinks that's a great name but because no one can do any better.

Some ordinary examples:

(1) a. If the goat is behind door #1, then the new car is behind door #2  
    b. If Eto'o regains his form, then Barça might advance  
    c. If Carl is at the party, then Lenny must also be at the party

Each of these is an ordinary indicative conditional sentence; two of them have epistemic modals in the conditional clause and all of them express a bit of

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What I am interested in is how well the indicatives play with the epistemic modals.

What these examples say is plain. Take (1b). This says that — within the set of possibilities compatible with the information at hand — among those in which Eto’o regains his form, some are possibilities in which Barça advance. Or take (1c). It says something about the occurrence of Lenny-is-at-the-party possibilities within the set of Carl-is-at-the-party possibilities — that, given the information at hand, every possibility of the latter stripe is also of the former stripe.

What sentences like these say is plain, how they say it isn’t. That’s my target here: How is it that the if’s in our examples manage to express conditional information and do so in a way compatible with how they play with epistemic modals?

The simplest story about how the if’s in our examples manages to express conditional information is that each of them expresses the information of a conditional. Which is to say: what these conditional sentences mean can be read-off the fact that if is a conditional operator. Which is to say: if is an operator, an iffy operator, and the same iffy operator in each of the sentences in (1). That is the gist of iffiness. But it is a hard line to maintain in the face of the data at hand: how conditional sentences play with modals seems to mean that if can’t be iffy.

Iffiness is an old school story about indicatives, and I want to defend one way of telling it. But not just any telling of it will do. So I want to show how our simple examples cause what looks like insurmountable trouble for old school views that take if to be iffy. Then, to make the point stick, I’ll sketch just how easy things would be — how easily the trouble could be surmounted — if we went new school and embraced Kratzer’s thesis that if is no operator at all, a fortiori not an iffy operator, and a fortiori not the same iffy operator in each of our example sentences it figures in. Instead, if simply restricts other operators. But the success of the restrictor analysis is no argument against Chuck Taylors and skyhooks tout court.

The old school story I favor is really a throwback. I say that if expresses a strict conditional operator over possibilities compatible with the context, and

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1We ought to be careful to distinguish between conditional sentences and conditional operators. The former are bits of natural language, the latter bits of some regimented language that may serve to represent the logical forms of the bits of natural language. But the point of carefully distinguishing is precisely so we can ignore the difference when it does not matter.

that it can do all the restricting that needs doing. But before saying how we can do this, I want to make it look for all the world like it can't be done.

2 Three facts

A story, old school or otherwise, has to answer to the facts. All the trouble for iffiness I am interested in can be got out of just three facts about how ifs and modals play together.3

I have lost my marbles. I know that one of them — Red or Yellow — is in the box. But I don’t know which. I find myself saying things like:

(2) Red might be in the box and Yellow might be in the box. So, if Yellow isn’t in the box, then Red must be; and if Red isn’t in the box, then Yellow must be

Any good story had better treat the ifs as consistent with the modal claims.

Fact 1. Conjunctions like these are relatively consistent:

a. might \( S_1 \) and might \( S_2 \)

b. if not \( S_1 \), then must \( S_2 \); and if not \( S_2 \), then must \( S_1 \)

That is: if (a) is consistent, then so is the conjunction of (a) and (b).

I do not know whether Carl made it to the party. But wherever Carl goes, Lenny is sure to follow. So if Carl is at the party, Lenny must be — Lenny is at the party, if Carl is. We just a glossed an if with a commingling must by an if with no (overt) modal at all. Thus:

(3) a. If Carl is at the party, then Lenny must be at the party \( \approx \)

b. If Carl is at the party, then Lenny is at the party

This pair has the ring of mutual entailers. And for good reason: (3a) looks to be true iff (3b) is.

3"Facts" is laying it on a little thick. The judgments are robust, I think, and the costs high for denying the facts as I put them. That’s all true even if what we may say about them is a matter for disputing. But it does not much matter: what I really care about is three characteristic seeming facts about ifs, mights, and musts that at first blush look like the kind of thing our best story ought to answer to. So let’s agree to take them at face value and see where that leads. Later, if your English breaks with mine or if your old school pride overwhelms, you can deny the facts or explain them away as your preferences dictate. That’s a lot of qualifiers, though, and "fact" is pithier anyway.
For suppose that such *if*’s validate a deduction theorem and modus ponens, and that *must* is factive. The left-to-right direction: assume that (3a) is true. And consider the argument:

\[(4) \text{ If Carl is at the party, then Lenny must be at the party; Carl is at the party; So: Lenny is at the party}\]

The first two sentences—intuitively speaking—entail the third. And that is pushed on us by the assumptions: from the first two sentences we have (by modus ponens) that *Lenny must be at the party*, which by factivity entails *Lenny is at the party*. Apply the deduction theorem and we have that *If Carl is at the party, then Lenny must be at the party* entails *If Carl is at the party, then Lenny is at the party*. Since we have assumed that (3a) is true, it then follows straightaway that (3b) must be. The right-to-left direction: assume that (3b) is true. And consider:

\[(5) \text{ If Carl is at the party, then Lenny is at the party; Carl is at the party; So: Lenny must be at the party}\]

This is as intuitive an entailment as we are likely to find. Whence it follows by the deduction theorem that *If Carl is at the party, then Lenny is at the party* on its own entails *If Carl is at the party, then Lenny must be at the party*. And so (3a) is true. And so if (3b) is true so must be (3a). Hence the ring of mutual entailment.

**Fact 2.** Conditional sentences like these are true in exactly the same scenarios:

- a. *if* \(S_1\), *then must* \(S_2\)
- b. *if* \(S_1\), *then* \(S_2\)

The glossing that this pattern of mutual entailment permits is a nifty trick. But that is only half the story. The interaction between *if* and *might* is different, and underwrites a different glossing by underwriting a different set of mutual entailments.

Alas, my team are not likely to win it all this year. It is late in the season and they have made too many miscues. But they are not quite out of it. If they win their remaining three games, and the team at the top lose theirs, they will be champions. But our last three are against strong teams and their last three are against cellar dwellers. Still, my spirits high: if they win out, they might win it all. Put another way, within the (relevant) my-team-wins-out
possibilities — of which there are some — lies a my-team-wins-it-all possibility; there is a my-team-wins-out possibility that is a my-team-wins-it-all possibility. But that is just to say that there are (relevant) my-team-wins-out-and-wins-it-all possibilities. Maybe not very many, and maybe not so close, but some.

Apart from keeping hope alive, the example also illustrates the mundane fact that we can gloss an indicative with a commingling epistemic might by a conjunction under the scope of might:

(6) a. If my team wins out, they might win it all ≈
   b. It might turn out that my team wins out and wins it all

That gloss sounds pretty good. And for good reason: conjunctions that would have to be happy if the truth of (6a) and (6b) could come apart are not happy at all:

(7) a. #If my team wins out, they might win it all; but they can’t win out and win it all
   b. #It might turn out that my team wins out and wins it all, but there’s no way that if they win out, they might win it all

That gives us the third fact about how if’s play with modals.4

**Fact 3.** Sentences like these are true in exactly the same scenarios:

a. if \( S_1 \), then might \( S_2 \)
   b. it might be that \([S_1 \text{ and } S_2]\)  

It’s now a matter of telling some story, iffy or otherwise, that answers to these

4There is a wrinkle: Fact 3 implies that if \( S_1 \), then might \( S_2 \) is true in just the same spots as if \( S_2 \), then might \( S_1 \). Seems odd:

(i) a. If I jump out the window, I might break a leg
   b. If I break a leg, I might jump out the window

The first is true, the second an overreaction. I intend, for now, to sweep this under the same rug that we sweep the odd way in which Some smoke and get cancer/Some get cancer and smoke don’t feel exactly equivalent even though some is a symmetric quantifier if ever there was one. See, e.g., the discussion of contraposition of bare conditionals in von Fintel (1997).
facts well enough.

3 Ground rules

Let’s simplify. Assume that meanings get associated with sentences by getting associated with formulas in an intermediate language that represents the relevant logical forms (LFSs) of them. Thus a story, old school or otherwise, has to first say what the relevant LFSs are and then assign those LFSs semantic values.

Suppose that conditional sentences do their information carrying by sporting conditional operators in their LFSs. So let \( L \) be the relevant intermediate language, generated from a stock of atomic sentence letters, negation (\( \neg \)), and conjunction (\( \land \)) in the usual way. But \( L \) also has the connective \( (if \cdot)(\cdot) \), and the modals \textit{must} and \textit{might}. What I have to be say can be said about an intermediate language that allows that the modals mix freely with the formulas of the non-modal fragment of \( L \) but restricts \( (if \cdot)(\cdot) \) so that it takes only non-modal sentences in its first argument and does not itself occur in the scope of negation. So assume that \( L \) is such an intermediate language. When these restrictions outlive their utility, we can exchange them for others.\(^5\)

Iffy stories take \( if \) to be a connective, an iffy connective, and the same iffy connective in each of our target examples. Since those sentences involve an epistemic modal — possibly covert or null, depending on taste — commingling with an \( if \), scope issues have to be sorted out. Take a sentence of the form

\[
\text{If } S_1 \text{ then MODAL } S_2
\]

and let \( S'_1 \) (\( S'_2 \)) be the \( L \)-representation for sentence \( S_1 \) (\( S_2 \)), and \textit{modal} the \( L \)-representation for \textit{MODAL}. We have a short menu of options for the relevant LFs for such a sentence — either the narrowscoped (\( 9a \)) or the widescoped (\( 9b \)):

\[
\begin{align*}
9a & \quad (if \ S'_1)(modal \ S'_2) \\
9b & \quad modal \ (if \ S'_1)(S'_2)
\end{align*}
\]

If you want to put your LFs in tree form, be my guest: opting for narrowscoping means opting for sisterhood between \textit{MODAL} and \( S_2 \); opting for widescoping means opting for sisterhood between \textit{modal} and \( S_2 \).

\(^5\)Conventions: \( p,q,r,\ldots \) range over sentences of \( L \) (subject to our constraints on \( L \)); \( i,j,k,\ldots \) range over worlds; and \( P,Q,R,\ldots \) range over sets of worlds. If \( p \)'s denotation is invariant across contexts - if \( [p]^c = [p]^{c'} \) no matter the choice for \( c \) and \( c' \) - let’s agree to conserve a bit of ink and sometimes omit the superscript; so, e.g., the ‘ifs’ I am focusing on here have non-modal antecedents, and so those antecedents will be context-invariant. And let’s not fuss over whether what is at stake is the ‘\( if \)’ of English or the ‘\( if \)’ of \( L \); context will disambiguate.
means opting for sisterhood between MODAL and \textit{if} $S_1$ \textit{then} $S_2$.

Those are the scope choices facing fans of iffiness. There are also choices about just which conditional operator we can say that the \textit{if} of English means. But our choices here are not completely free, and some ground rules will impose some order on what we may say. These will constrain our choice by saying what must be true of the kinds of conditional operators rightfully so called. But before getting to that, I’ll start with what I will assume about contexts and the standard quantificational view of the modals.

First, a general constraint: assume that truth-values — for our modal and \textit{if}-constructions, as well as for the boolean fragment of $L$ — are assigned at an index (world) $i$ with respect to a context. I will assume that $W$, the space of possible worlds, is finite. Nothing important turns on this, and it simplifies things.

For the boolean bits, contexts are idle. It is the job of the modals to quantify over possibilities compatible with the context and it is the job of context to select the domains over which the modals do their job. What I want to say can be said in a way that is agnostic about just what kinds of things contexts are: all I insist is that, given a world, they determine a set of possibilities that modals at that world quantify over. The functions doing the determining need to be well-behaved; \textit{might} and \textit{must} quantify over the sets of worlds so determined and have their standard quantificational force.

Where $c$ is a context — replete with whatever things contexts are replete with — an epistemic modal base $C$ determined by it is just what we need:

\begin{equation}
C = \lambda i. \{ j : j \text{ is compatible with the } c\text{-relevant information at } i \}
\end{equation}

Since the only context dependence at stake will be dependence on such bases, we can get by just as well by taking them to go proxy for \textit{bona fide} contexts, granting them the honorific “contexts”, and relativizing the assignment of truth-values to index-modal base pairs directly. No harm comes from that, and it makes for a prettier view.

But not just any function from indices to sets of indices will do as a (proxy) context. So the first constraint is that $C$ be well-behaved — that is, reflexive and

\[\text{reflexive}
\]

\[\text{Symmetric}
\]

\[\text{Transitive}
\]

\[\text{Well-behaved}
\]

The problems and prospects for iffiness are independent of just whose information in a context — speaker, speaker plus hearer, just the hearer, just the hearer’s picture of what the speaker intends, and so on — counts for selecting the domains for the modals to do their job, and whether or not that information is information-at-a-context at all. So let’s keep things simple here. If you’d rather be reading a paper which has these (and other) complexities at the forefront, see von Fintel and Gillies (2007a,b,c) and the references therein.
euclidean:

(W) WELL-BEHAVEDNESS
   a. \( i \in C_i \) (REFLEXIVENESS)
   b. if \( j \in C_i \) then \( C_i \subseteq C_j \) (EUCLIDEANNESS)

If \( C \) is well-behaved then \( C_i \) is closed — well-behavedness implies that if \( j \in C_i \), then \( C_j = C_i \).

Proof. Suppose \( j \in C_i \). Consider any \( k \in C_j \). Since \( C \) is euclidean and \( j \in C_i \), \( C_i \subseteq C_j \). Since \( C \) is reflexive, \( i \in C_i \) and thus \( i \in C_j \). Appeal to euclideanness again: since \( i \in C_j \), \( C_j \subseteq C_k \) and hence \( i \in C_k \). And once more: since \( i \in C_k \), \( C_k \subseteq C_i \). And now reflexiveness: \( k \in C_k \) and so \( k \in C_i \). (The inclusion in the other direction just is euclideanness.)

Modals have their usual quantification oomph: must (at \( i \), with respect to \( C \)) acts as a universal quantifier, and might an existential quantifier, over \( C_i \).

(M) MODAL FORCE
   a. \( [\text{might} \ p]^{C,i} = 1 \) iff \( C_i \cap [p]^C \neq \emptyset \)
   b. \( [\text{must} \ p]^{C,i} = 1 \) iff \( C_i \subseteq [p]^C \)

Putting (W) and (M) together: whatever is the case at \( i \) is relevant, though perhaps not decisive, to the truth of a modal claim at \( i \) (reflexiveness); and if a possibility is compatible with the context, then that is transparent in the context (euclideanness).

4 Conditional operators

By saying something about what must be true of an operator for it to be a conditional operator properly so called we thereby say something about what must be true for a story to be iffy. Iffiness begins with a simple thought: conditional sentences say what might or must be if such-and-such is or turns out to be the case by expressing a conditional operator. We can get all we need out of just that little slogan.

First: in the cases we’ll care about, if’s don’t take a stand on whether such-and-such is the case and so they are typically happiest being uttered in circumstances in which such-and-such is compatible with the context as it
stands when the conditional is issued. I will take it as a definedness condition on the semantics for conditionals that they are happily issued.\footnote{The requirement isn’t novel. See, e.g., \textcite{Stalnaker1975}; \textcite{Fintel1998}; \textcite{Gillies2004}.}

(D) \textbf{DEFINEDNESS}

\begin{enumerate}
\item \((p \rightarrow q)\) is appropriate (at \(i\), in \(C\)) only if \(p\) is compatible with \(C_i\).
\item \([[(p \rightarrow q)]^C]_i\) is defined only if \((p \rightarrow q)\) is appropriate (at \(i\), in \(C\)).
\end{enumerate}

This is still slippery—I have said nothing about what counts as compatibility, and haven’t said what happens when an inappropriate \(i\) is issued — but it will do.

Second: a conditional expresses a relation between antecedent and consequent. That is: \(i\) expresses a relation between the set of antecedent possibilities and the set of consequent possibilities. Take an arbitrary conditional like \((p \rightarrow q)\) at \(i\), in \(C\). And let \(P\) and \(Q\) be the sets of antecedent and consequent possibilities so related by the \(i\). For this to be a conditional operator properly so called, what this expresses must be a relation \(R\) between \(P\), perhaps plus some domain \(D_i\), and \(Q\).

\(D_i\) is the set of possibilities relevant for the \(i\) at \(i\). Although I have put that as a function of \(i\), depending on your favorite theory \(D_i\) may be a function of \(i\), of \(C\), of \(p\), of \(q\), or of your kitchen sink. We will return to that shortly. No matter your favorite theory, we can still \textit{ex ante} agree to this much: \(i\) is always among the possibilities relevant for an \(i\) at \(i\), and \textit{only} possibilities compatible with the context are relevant for an \(i\) at \(i\). That is: \(i \in D_i\) and \(D_i \subseteq C_i\). The first is a platitude, the second means that an \(i\) at \(i\) is supposed to say something about the possibilities compatible with \(C\).

The requirement then is that a conditional operator can be properly so called only if its truth-conditions can be put this way:

\begin{equation}
\text{if defined, } [[(p \rightarrow q)]^C]_i = 1 \text{ iff } R(D_i \cap P, Q) \tag{11}
\end{equation}

for some set of possibilities \(D_i\) and relation \(R\), where \(i \in D_i\) and \(D_i \subseteq C_i\).

Third: but not just any relation between \(D_i \cap P\) and \(Q\) counts as a relation that a conditional properly so called could express. I insist on three minimal constraints. (1) That \(D_i \cap P\) imposes some order on the set of \(Q\)’s so related. (2) That \(Q\) matters to whether the relation holds. (3) That — plus or minus just a bit — only the relationship between the possibilities in \(D_i \cap P\) and the possibili-
ties in $Q$ matter to whether the relation holds. These are not controversial, but do bear some unpacking.

The order imposed by the antecedent:

(C1) $R$ is something $(if \cdot) (\cdot)$ at $i$ could mean only if:
   a. REF: $R(D_i \cap P, P)$
   b. MON: $R(D_i \cap P, Q)$ and $Q \subseteq S$ imply $R(D_i \cap P, S)$
   c. CON: $R(D_i \cap P, Q)$ and $R(D_i \cap P, S)$ imply $R(D_i \cap P, Q \cap S)$

If an argument is wanted for so constraining $R$, it is this: such $R$’s are precisely those for which the set of $Q$’s a $D_i \cap P$ bears it to form an ideal that contains $P$. If another argument is wanted, it is this: such relations jointly characterize the basic conditional logic.\(^8\)

(C2) $R$ is something $(if \cdot) (\cdot)$ at $i$ could mean only if:
   
   if $D_i \cap P \neq \emptyset$ then there is a $Q$ and $Q'$ such that: $R(D_i \cap P, Q)$ but not $R(D_i \cap P, Q')$

This means that $R$ cares about how $D_i \cap P$ relates to $Q$. So long as there are some relevant possibilities, there have to be some $Q$’s for which the relation holds and some for which it doesn’t. (When put as a property of quantifiers, this is usually called \textsc{activity}.)

And finally: $R$ is a relation between the sets of possibilities. Thus if $R$ holds at all between $D_i$-plus-the-antecedent-determined-$P$ and the consequent-possibilities $Q$, $R$ will hold between any two sets of things that play the right possibility role. The idea is simple, the execution harder. That is because to say this properly, we have to know more about what goes in to determining $D_i$, and that will vary depending on your favorite story about \textit{if}.

Three examples: (1) Suppose your favorite story takes \textit{if} to be a variably strict conditional (Stalnaker, 1968; Lewis, 1973). For every world $i$, let $\preceq_i$ be an ordering of worlds, a relation of comparative similarity (at least) weakly centered on $i$. Given a conditional $(if p) (q)$ at $i$ in $C$, you will want to identify $D_i$ with the set of possibilities no more dissimilar than the most similar $p$-world to $i$. (2) Perhaps your favorite Lewis-inspired story comes not from D.K. but from C.I. You thus take \textit{if} to be strict implication. Then you will want to identify $D_i$ instead with the set $W$ of all possibilities. But that, too, can be put

\(^{8}\)See Veltman (1985) for a proof.
in terms of orderings: your ordering \( \preceq_i \) is universal, treating all worlds the same. Whence it follows that — since the nearest \( p \)-world is the same distance from \( i \) as is every world — taking \( D_i \) to be the set of possibilities no further from \( i \) as the nearest \( p \)-world amounts to taking \( D_i \) to be the set of all worlds \( W \). (3) Suppose you are smitten by truth-tables, and your favorite story about indicatives is the material conditional story. You will then want to take \( D_i \) to be \( \{i\} \). Equivalently: you will have a maximally discerning ordering — every world, like no man, is an island — and take \( D_i \) to be the set of closest worlds to \( i \) simpliciter.

What is important is this: suppose your favorite story posits some additional structure to modal space to find just the right worlds which, when combined with \( P \), gives the set of worlds relevant for evaluating \( Q \). That means that your favorite story cares about how \( P \) relates to \( Q \) but also about the distribution of the worlds in \( P \) compared to the distribution in \( Q \) — perhaps insisting that it is the closest worlds in \( P \) to \( i \) that must bear \( R \) to \( Q \). If we systematically swap possibilities for possibilities in a way that we preserve the relevant structure, then the conditional relation ought to hold pre-swapping iff it holds post-swapping. And mutatis mutandis for \( D_i \): since once the posited structure does its job determining \( D_i \), then any systematic swapping of possibilities that leaves the domain untouched should also leave the conditional relation untouched.\(^9\)

If \( \pi \) is such a mapping and \( P \) a set of worlds, let \( \pi(P) \) be the set of worlds \( i \) such that \( \pi(j) = i \) for some \( j \in P \).

(C3) \( R \) is something \( (\text{if } \cdot)(\cdot) \) at \( i \) could mean only if:

\[
R(D_i \cap P, Q) \text{ implies } R(\pi(D_i \cap P), \pi(Q))
\]

This does generalize the familiar constraint on quantifiers — it allows conditional operators to care about both the relationship between \( P \) and \( Q \), but also where the satisfying worlds are. If \( \preceq_i \) is the universal ordering, so that \( D_i = W \), then this requirement reduces to the more familiar quantitative one. And if \( D_i = \{i\} \), it trivializes. (Hence this property is sometimes called QUALITY.)

I am insisting that a story is iffy only if the truth conditions for an indicative \( (\text{if } p)(q) \) at \( i \) in \( C_i \) can be put as a relation between \( R \) between \( D_i \cap P \)

\(^9\)This is, of course, a natural extension of the familiar requirement that quantifiers be quantitative: for \( Q \) to be a quantifier (with domain \( E \)) it must be that \( Q_{f(A,B)} \) iff \( Q_{f(f(A)),f(f(B))} \) where \( f \) is an isomorphism of \( E \). Once we have structure to our domain, this will not do. The more general constraint is then to require that \( Q \) be invariant under \( O \)-automorphisms of the domain, where \( O \) is the ordering that imposes the posited structure. We can get by with slightly less: namely, stability under \( D_i \)-invariant automorphisms.
and $Q$. Still, it seems open to take the conditional to be true just in case most/many/several/some/just the right possibilities in $D_i \cap P$ are in $Q$. But that is not so: given (C1)–(C3), if must express inclusion. That is: $R(D_i \cap P, Q)$ holds iff $D_i \cap P \subseteq Q$.

**Proof.** Assume that $R$ is a conditional relation. I care about the left-to-right direction.

Suppose — for reductio — that $R(D_i \cap P, Q)$ but $D_i \cap P \not\subseteq Q$. What we’ll see is: (i) $R(D_i \cap P, P \cap Q)$; (ii) the world that witnesses that $D_i \cap P \not\subseteq Q$ can be exploited by (C3) to show that no world in $P \cap Q$ plays a role in $R(D_i \cap P, P \cap Q)$ holding — from which it follows that $R(D_i \cap P, \emptyset)$; (iii) from which it follows that $D_i \cap P$ must be empty — a contradiction.

**Ad (i):** By hypothesis $R(D_i \cap P, Q)$. Since $R$ satisfies ref of (C1), $R(D_i \cap P, P \cap Q)$. Applying con of (C1):

$$R(D_i \cap P, P \cap Q)$$

**Ad (ii):** Let $j$ be a witness to $D_i \cap P \not\subseteq Q$. So $j \in D_i \cap P$ but $j \not\in Q$. Now pick any confirming instance $k$ — that is, any $k \in (D_i \cap P) \cap Q$ — and let $\pi$ be the mapping that swaps $k$ and $j$ and leaves all else untouched:

- $\pi(j) = k$
- $\pi(k) = j$
- $\pi(i) = i$ for every $i \notin \{j, k\}$

By (i) $R(D_i \cap P, P \cap Q)$. Hence, by (C3), $R(\pi(D_i \cap P), \pi(P \cap Q))$. But $\pi$ doesn’t affect $D_i \cap P$. So: $R(D_i \cap P, \pi(P \cap Q))$. That is: $R$ holds between $D_i \cap P$ and both $P \cap Q$ and $\pi(P \cap Q)$. Hence — by con of (C1) — it holds also between $D_i \cap P$ and their intersection: $R(D_i \cap P, (P \cap Q) \cap \pi(P \cap Q))$. But $\pi(P \cap Q) = ((P \cap Q) \setminus \{j\}) \cup \{k\}$, so their intersection is $(P \cap Q) \setminus \{j\}$. So: $R(D_i \cap P, (P \cap Q) \setminus \{k\})$. Which is to say that $k$ is irrelevant for $R$’s holding. But $k$ was any world in $(D_i \cap P) \cap Q$, so finiteness plus con of (C1) implies $R(D_i \cap P, \emptyset)$.

**Ad (iii):** $R$ satisfies mon of (C1). Since $R(D_i \cap P, \emptyset)$, it holds that for any $S$ whatever $R(D_i \cap P, S)$. Whence it follows from (C2) that $D_i \cap P = \emptyset$. And that contradicts the assumption that $D_i \cap P \not\subseteq Q$.

The intuitive version is just this: if $R$ holds between $D_i \cap P$ and $Q$ then the former must be included in the latter. That is because if things didn’t go that way then the witnessing counterexample world could play the role of any one

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10This was first proved by van Benthem — see, e.g., *van Benthem (1986)*. The version I give is simpler (we’re ignoring the infinite case) and a bit more general (slightly weaker assumptions); the proof is based on one in *Veltman (1985)*, but generalizes it to cover the case of ordering semantics for conditional operators.
of the confirming worlds. But that would mean that confirming worlds play no role. Nothing like that could be something a conditional properly so called could mean. So $D_i \cap P$ must be included in $Q$ after all.

5 Scope matters

Assume, as old school stories have it, that if is a conditional operator. Since we are interested in cases in which the if commingles with a modal, we have to decide whether the modal has narrow or wide scope. The trouble is that, since if expresses inclusion, neither will do. That is because our story also has to answer to the facts. One choice for scope relations seems ruled out by the consistency facts (Fact 1), the other by the entailments (Facts 2 and 3). And that looks like pretty bad news for old school iffiness.

For suppose we plump for narrowscoping. Then, given the ground rules, we cannot predict the consistency of the likes of (2).

That stretch of discourse begins with a modal claim the red marble might be in the box and the yellow marble might be in the box and goes from there to two if’s. Narrowscoping in $L$

(12) a. Modal claim:
   (i) Red might be in the box and Yellow might be in the box
   (ii) $\text{might } p \land \text{might } q$

b. First conditional:
   (i) If Yellow isn’t in the box, then Red must be
   (ii) $(\text{if } \neg q)(\text{must } p)$

c. Second conditional:
   (i) If Red isn’t in the box, then Yellow one must be
   (ii) $(\text{if } \neg p)(\text{must } q)$

Any good story has to allow that the bundle of if’s in (12b) and (12c) is consistent with the modal claim in (12a). But this — even modulo a choice for $D_i$ — seems to be beyond what can be delivered by an iffy story.

Proof. For suppose otherwise — that the regimented formulas in $L$ are all true at a world, say $i$, with respect to $C$. Just one of my marbles is in the box. So any world in $C_i$ is either a $p$-world or a $q$-world, but not both; $C$ is well-behaved, so $i \in C_i$. Thus: either (1) $i \in \llbracket \neg q \rrbracket$, or (2) $i \in \llbracket \neg p \rrbracket$. 


If (1): By hypothesis \( \llbracket (if \neg q)(must p) \rrbracket \subseteq \llbracket must p \rrbracket \). Since \( i \in D_i \), it then follows that \( i \in \llbracket must p \rrbracket \) — which is to say \( \llbracket must p \rrbracket^{C,i} = 1 \). Thus \( C_i \) has only \( p \)-worlds in it. But that is at odds with the second conjunct of (12a): its truth at \( i \) guarantees a \( q \)-world, hence a \( \neg p \)-world, in \( C_i \).

If (2): By hypothesis \( \llbracket (if \neg p)(must q) \rrbracket \subseteq \llbracket must q \rrbracket \). Since \( i \in D_i \), it then follows that \( i \in \llbracket must q \rrbracket \) — which is to say \( \llbracket must q \rrbracket^{C,i} = 1 \). Thus \( C_i \) has only \( q \)-worlds in it. But that is at odds with the first conjunct of (12a): its truth at \( i \) guarantees a \( p \)-world, hence a \( \neg q \)-world, in \( C_i \).

We should *ceteris paribus* go with narrowscoping. But, at least so it appears, *ceteris* are not very *paribus*. So suppose instead that commingling modals scope over the *if*-constructions in which they occur. Now it is the mutual entailments — as in (3) and (6) — that cause trouble. Widescoping in \( L \):

\[(13) \]

a. For (3a):
   i. If Carl is at the party, then Lenny must be at the party
   ii. *must* *(if \( p \))*(\( q \))

b. For (3b):
   i. If Carl is at the party, then Lenny is at the party
   ii. *(if \( p \))*(\( q \))

\[(14) \]

a. For (6a):
   i. If my team wins out, they might win it all
   ii. *might* *(if \( p \))*(\( q \))

b. For (6b):
   i. It might turn out that my team wins out and wins it all
   ii. *might* *(\( p \land q \))*

What is needed is a semantics for the conditional operator *(if \( \cdot \))*(\( \cdot \)) that can predict both patterns. But — assuming widescoped *if*s — paths that might lead to one pretty reliably lead away from the other.

So far I have insisted that \( i \) is always among the relevant worlds to an *if* at \( i \) \( (i \in D_i) \) and also that only worlds compatible with the context are relevant \( (D_i \subseteq C_i) \). Here I am in good company. But perhaps there is even more interaction between domains and contexts.

Call a theory *egalitarian* if it requires domains to be invariant across worlds compatible with a context — if whenever \( j \in C_i \) then \( D_j = D_i \). That means that distinctions made by \( D \)'s are unaffected when those distinctions are made
iffiness

from behind a veil of ignorance. Chauvinistic theories are those that are not egalitarian, allowing differences from behind the veil to matter to what possibilities get selected for domainhood, and thus allowing the possibility that a $j \in C_i$ be such that $D_j \neq D_i$. Once we have agreed that, for any $i$, $D_i$ selects from the worlds compatible with $C$ and must include $i$, it is a further question whether we want to be egalitarians or chauvinists.

History is littered with chauvinists. The material conditional analysis says that the only possibility relevant for the truth of an if at $i$ in $C$ is $i$ itself. And similarly for an if at $j$—there only $j$ matters. Thus, except in the odd case where the context rules out uncertainty altogether, we will have that $D_j \neq D_i$, for any choice of $i$ and $j$ compatible with $C$. But it is hard to be a chauvinist. That is because, assuming the particulars of the chauvinistic theory are compatible with there being a $(p \land \lnot q)$-world in $C_i$ but not in $D_i$, no such story will render $(\text{if} \ p)(q)$ and must $(\text{if} \ p)(q)$ mutual entailers. And so—given widescoping—no such story will predict entailments like those in (13) and so will have a hard time with Fact 2.

For consider a $(p \land \lnot q)$-world—call it $j$—and suppose that $C_i$ does, but $D_i$ does not, contain $j$. Then every possibility in $D_i \cap [p]$ is in $[q]$ and the plain if is true (at $i$, in $C$): $[[\text{if} \ p](q)]^{C,i} = 1$. But not the widescoped if. That is because there is a world in $C_i$—namely $j$—such that not every possibility in $D_j \cap [p]$ is a possibility in $[q]$. Thus $[[\text{if} \ p](q)]^{C,j} = 0$ and so it is not true that the plain if is true at every world in $C_i$ and so $[[\text{must} \ (\text{if} \ p)(q)]^{C,i} = 0$.

Egalitarianism is no walk in the park either. But here it is (14) that is trouble. Note, first, that egalitarianism implies that $D_i$ covers $C_i$ completely—that $D_i = C_i$.

For consider a $(p \land \lnot q)$-world—call it $j$—and suppose that $C_i$ does, but $D_i$ does not, contain $j$. Then every possibility in $D_i \cap [p]$ is in $[q]$ and the plain if is true (at $i$, in $C$): $[[\text{if} \ p](q)]^{C,i} = 1$. But not the widescoped if. That is because there is a world in $C_i$—namely $j$—such that not every possibility in $D_j \cap [p]$ is a possibility in $[q]$. Thus $[[\text{if} \ p](q)]^{C,j} = 0$ and so it is not true that the plain if is true at every world in $C_i$ and so $[[\text{must} \ (\text{if} \ p)(q)]^{C,i} = 0$.

Proof. Assume otherwise. $D_i \subseteq C_i$, so there must be a $j \in C_i$ such that $j \notin D_i$. Egalitarianism gives us that $D_j = D_i$. But we know that $j \notin D_j$. Contradiction. □
must \((p \supset q)\) which, given \((W)\), is equivalent to must must \((p \supset q)\). And that, in turn, is equivalent to must \((if \ p) (q)\).\(^{12}\)

But there is still trouble afoot. From this degree of fit between \(D_i\) and \(C_i\) it follows straightaway that no two possibilities compatible with \(C\) can differ over an if issued in \(C\). That is:

\[ (15) \quad \llbracket (if \ p) (q) \rrbracket^{C,i} = 1 \text{ iff for every } j \in C_i : \llbracket (if \ p) (q) \rrbracket^{C,j} = 1 \]

**Proof.** \(\llbracket (if \ p) (q) \rrbracket^{C,i} = 1 \) iff \(D_i \cap \llbracket p \rrbracket \subseteq \llbracket q \rrbracket\). Given egalitarianism: iff, for any \(j \in C_i, D_j \cap \llbracket p \rrbracket \subseteq \llbracket q \rrbracket\). Equivalently: iff, for any \(j \in C_i, C_j \cap \llbracket p \rrbracket \subseteq \llbracket q \rrbracket\) — that is, iff for every such \(j, \llbracket (if \ p) (q) \rrbracket^{C,j} = 1\). \(\square\)

But any story with this equivalence — when coupled with widescoping — will have a hard time with entailments like those in \((14)\) and so will have trouble with Fact \(3\). That is because \((15)\) together with \((M)\) implies

\[ (16) \quad \llbracket might \ (if \ p) (q) \rrbracket^{C,i} = 1 \text{ iff } \llbracket must \ (if \ p) (q) \rrbracket^{C,i} = 1 \]

**Proof.** Note that \(\llbracket might \ (if \ p) (q) \rrbracket^{C,i} = 1 \) iff the plain conditional \((if \ p) (q)\) is true somewhere in \(C_i\). But by \((15)\) the plain if is true somewhere in \(C_i\) iff it is true everywhere in \(C_i\). And it is true everywhere in \(C_i\) just in case \(\llbracket must \ (if \ p) (q) \rrbracket^{C,i} = 1\). \(\square\)

When coupled with widescoping, such a story can account for the mutual entailment between \((14a)\) and \((14b)\), but only by trivializing: \(might \ (p \land q)\) entails \(must \ (if \ p) (q)\). Not a happy result.

Iffiness requires conditionals, if they are to be properly so called, to have a structure that does not play nice with modals. For no way of resolving the relative scopes will work, and there is no good reason to think we should settle

\(^{12}\) For the record: (i) Stalnaker (1975) is not officially agnostic about chauvinism, but that is only because he there requires that \(\leq_i\) induce a total order (that's the assumption otherwise known as the Uniqueness Assumption). But the pragmatic mechanisms he develops there are agnostic on the chauvinism question — what he says about how the context constrains selection functions is compatible with both. (ii) I myself see every reason for egalitarianism and none for chauvinism — for one thing that makes for an easy explanation of why negated ifs sounds so much like a conjunction of antecedent and negated consequent all under the scope of might. (iii) I like the result that egalitarianism implies strictness for if, since that is an upshot I like anyway — for more on context-dependent strictness see, e.g., von Fintel (1998, 2001); Gillies (2004, 2007a,b).
for anything less than a uniform story. That is pretty bad news for iffiness.

6 Iffiness lost

Lewis (1975) famously argued that if’s appearing in certain quantificational constructions (under adverbs of quantification) are not properly iffy, that the if in

\[
\begin{cases}
\text{Always} \\
\text{Sometimes} & \text{if a man owns a donkey, he beats it} \\
\text{Never}
\end{cases}
\]

is not a conditional operator but instead acts as a non-connective whose only job is to mark an argument-place. The relevant structure is not a \(Q\)-adverb scoped over a conditional, he said, but something like

\(Q\)-adverb + if-clause + then-clause

The job of the if-clause is to restrict the domain over which the adverb (unselectively) quantifies, and allegedly that restricting job is a job that cannot be done by treating if as a conditional connective. If \(Q\)-adverb is universal, maybe an iffy if will work; but if it is existential, then conjunction does better. I want to set the issue about adverbial (and adnomial, for that matter) quantifiers aside for two reasons. First because I doubt the allegation sticks.\(^{13}\) But that is another argument for another day. And second because it will do us good to focus on simple cases.

Still, the trouble for iffiness that is center stage here is quite a lot like this. We have to make room for interaction between if-clauses and the domains our modals quantify over. But that interaction is tricky. That is because it looks impossible to assign if the same conditional meaning — thereby taking its contribution to be an iffy one — in all of our examples. Indeed, when the modal is universal a conditional relation looks good; but when the modal is existential, conjunction looks better.

This is pretty much the same trouble Lewis saw for if’s occurring under adverbs of quantification, and led him to conclude that such if’s are non-connectives. Just as with adverbial quantifiers, there is a fast and easy solution

\(^{13}\)There are ways to get the jobs done after all: see, e.g., Belnap (1970); Dekker (2001); von Fintel and Iatridou (2003).
to the problem if we get rid of the old school idea that *if* is a conditional connective and plump instead for anti-iffiness.\textsuperscript{14}

The most forceful way of putting the anti-iffy thesis is Kratzer’s:

The history of the conditional is the history of a syntactic mistake. There is no two-place “if...then” connective in the logical forms for natural languages. “If”-clauses are devices for restricting the domains of various operators. Kratzer (1986, p. 11)

Instead of searching for a conditional operator properly so called that contributes the same bit whether it commingles with a modal or not, we search for an operator for *if* to restrict.

And, for indicative conditionals, we do not have to search far: the operators are (possibly covert) epistemic modals. Take a conditional sentence of the form

\begin{equation}
\text{If } S_1 \text{ then } S_2
\end{equation}

Its LF does not have some conditional operator linking $S_1$ and $S_2$. It’s a modal, expressing some quantificational force $Q$ and saying that among the (relevant) $S_1$-worlds, $Q$ of them are $S_2$-worlds. The modal is the operator, the *if*-clause restricts its domain of quantification.\textsuperscript{15} When no overt modal is present — as in (16) — take it to be *must*.

So it is the modals, not *ifs*, that have a bit more structure than we thought. They have logical forms along the lines of $\text{modal}(p)(q)$, with the usual quantificational force:

\begin{itemize}
\item[(M')] **MODAL FORCE, AMENDED**
\begin{itemize}
\item[a.] if defined, $\llbracket \text{might} \ (p) \ (q) \rrbracket^{C,w} = 1 \ \text{iff} \ (C_i \cap \llbracket p \rrbracket^C) \cap \llbracket q \rrbracket^C \neq \emptyset$
\item[b.] if defined, $\llbracket \text{must} \ (p) \ (q) \rrbracket^{C,w} = 1 \ \text{iff} \ (C_i \cap \llbracket p \rrbracket) \subseteq \llbracket q \rrbracket^C$
\end{itemize}
\end{itemize}

This plus two assumptions buys an anti-iffy semantics that easily squares with both the consistency facts (Fact 1) and the mutual entailments (Facts 2 and 3).

First: assume that when no restrictor is explicit — as in *Blue might be in the box* or *Yellow must be in the box* — the first argument in the LF of the modal is filled by your favorite tautology ($\top$). In those cases there is nothing to choose


\textsuperscript{15}That means opting for sisterhood between $\text{modal}$ and $S_2$, and dominance between $\text{modal}$ and *if* $S_1$. 
between an analysis that follows (M) and an analysis that follows (M'), and so the latter generalizes the former.

Second: assume that the job of if-clauses is to make a (non-trivial) restrictor explicit. Since we are supposing that plain if's — those without an explicit commingling modal — carry a covert must, that is about all we need to predict that stretches of conditional and modal talk like that in (2) are consistent and the mutual entailments like those in (3) and (6).16

Collecting the pieces:

(17) ANTI-IFFY STORY
   a. a sentence of the form if $S_1$ then $S_2$ has LF:
      (i) $\text{MODAL} (S'_1)(S^*)$ if $S_2 = \text{MODAL} S^*$
      (ii) $\text{must} (S'_1)(S'_2)$ otherwise
   b. interpret LFs as in (M')

(Here $S'_1 (S'_2)$ is the LF for sentence $S_1 (S_2)$.)

Return to the case of the missing marbles. Taking the if-clauses to be restrictors:

(18) a. Modal claim:
   (i) Red might be in the box and Yellow might be in the box
   (ii) \( \text{might} (\top)(p) \land \text{might} (\top)(q) \)
   b. First “conditional”:
   (i) If Yellow isn’t in the box, then Red must be
   (ii) $\text{must} (\neg q)(p)$
   c. Second “conditional”:
   (i) If Red isn’t in the box, then Yellow must be
   (ii) $\text{must} (\neg p)(q)$

It’s modals all the way down.

It is easy enough to see how these can all be true. I am in $i$ and there are just two worlds compatible with the facts I have, $i$ and $j$. The first is a \((p \land \neg q)\)-world, the second a \((q \land \neg p)\)-world. The restrictors in (18a) are trivial, so it is true at $i$ iff $C_i$ has a $p$-world in it and a $q$-world in it; $i$ witnesses the

16Equivalently, we could treat all if's as implicitly modal: if $S_1$ then $S_2$ invariably gets an analysis along the lines of $\text{must} (S'_1)(S'_2)$. If $S_2$ happens to carry a modal — in particular if $S_2$ has as LF $\text{modal} (\top)(r)$ — then the well-behavedness of $C$ guarantees that $\text{must} (S'_1)(S'_2)$ is true at $i$ in $C$ iff $\text{modal} (S'_1)(r)$ is. The inner modal carries the day, but under the restriction imposed by the outer modal. This is equivalent to what Geurts (2005) calls “fusing” of the two modals.
Iffiness

first conjunct, \( j \) the second. The restricting \( if \)-clause of (18b) makes sure that the \emph{must} ends up quantifying only over the \( \neg q \)-worlds compatible with \( C \): (18b) is true at \( i \) iff all of the worlds \( C_i \cap \llbracket \neg q \rrbracket \) are \( p \)-worlds. And the only one, \( i \), is. Similarly for the \emph{must} in (18c): it quantifies over the \( \neg p \)-worlds in \( C_i \), checking to see that they are all \( q \)-worlds.

It is just as easy to square this picture with Facts 2 and 3. Take the interaction of \emph{if} and \emph{must} in (3). The two conditionals \emph{If Carl is at the party, then Lenny must be at the party} and \emph{If Carl is at the party, then Lenny is at the party} have the same LF: \( \text{must} \ (p)(q) \). It would thus be hard, and pretty undesirable, for them not to entail one another.

And the interaction of \emph{if} and \emph{might} in (6):

(19) a. For (6a):
1. If my team wins out, they might win it all
2. \( \text{might} \ (p)(q) \)

b. For (6b):
1. It might turn out that my team wins out and wins it all
2. \( \text{might} \ (\top)(p \land q) \)

If (19b) is true at \( i \) in \( C \) then \( C_i \) has a \( (p \land q) \)-world in it. But then that same world must be in \( C_i \cap \llbracket p \rrbracket \). It is a \( q \)-world, and that will witness the truth of (19a) at \( i \). Going the other direction: if (19a) is true at \( i \) in \( C \), then there are some \( q \)-worlds in \( C_i \cap \llbracket p \rrbracket \). Any one of those will do as a \( (p \land q) \)-world in \( C_i \), and that is sufficient for (19b) to be true at \( i \).

These explanations are wicked easy. And, given the trouble for iffiness, it looks like the only game in town is to deny that \emph{if} is a conditional operator in the first place. That stings.

7 Iffiness regained

The problem for iffiness is that there is an interaction between \( if \)-clauses and the domains our modals quantify over. That is an interaction that seems hard to square with the thesis that \emph{if} is a binary operator with a conditional meaning if we assume that it has the same meaning in each of the cases we care about here.

But we have overlooked a possibility. We insisted that for a story to be iffy it must say that \( (if \ p)(q) \) at \( i \) in \( C \) expresses some relation \( R \) between \( D_i \cap P \)
and $Q$, where $P$ and $Q$ are the sets of antecedent and consequent worlds. That
is all right. But we unthinkingly assumed that the context relevant for figuring
out what these worlds are must always be $C$ just because that was the context
as it stood when the if was issued. That was a mistake, and setting it straight
sets the record straight for old school iffiness.

The Ramsey test — the schoolyard version, anyway — says that a conditional
if $S_1$ then $S_2$, uttered against some background context, is true just in case
adding the information carried by $S_1$ to that context lands you in a spot in
which $S_2$ is true. That is pretty much right. But since truth depends on context
and index, that means that “adding the information carried by $S_i$” has two jobs
to do and we have to keep track of how both get done. One job is to restrict
the set of possibilities throughout which we check for $S_2$’s truth. We check
to see whether, for every (relevant) possibility in which $S_1$ is true, $S_2$ is true.
That is the index-shifting job. The other job is to contribute to the background
context against which we check for $S_2$’s truth. When we check whether $S_2$ is
true at some $S_1$-possibility, we do so not with respect to the context as it was
when the conditional was issued, but the derived or subordinate context got by
hypothetically adding the information carried by the antecedent to it. That is
the context-shifting job.

Here is the simplest way to keep track of both jobs if-clauses do:

\[(20)\]  

\[
\text{IFFINESS + SHIFTINESS}
\]

\[a. \] if defined, $\llbracket (\text{if } p)(q) \rrbracket^C_i = 1$ iff $C_i \cap \llbracket p \rrbracket^C \subseteq \llbracket q \rrbracket^{C+p}$

\[b. \] $C + p = \lambda i.C_i \cap \llbracket p \rrbracket^C$

Such a story about if is iffy: if expresses is a relation between relevant an-
tecedent and consequent worlds. And it is doubly shifty. It is index-shifty since
the truth of $(\text{if } p)(q)$ at $i$ depends on the truth of the constituent $q$ at worlds
other than $i$. It is context-shifty since the truth of $(\text{if } p)(q)$ in $C$ depends on
the truth of the constituent $q$ in contexts other than $C$.

The difference between interpreting $q$ against the backdrop of the prior
context $C$ and against the backdrop of $C + p$ is a difference that makes no
difference if $q$ has no context sensitive bits in it. But if $q$ does have context
sensitive bits in it — if it has some bit, like might or must, whose semantic
value depends non-trivially on $C$ — then this is a difference which makes all
the difference. For example: consider a modal like must $q$. Then $C$ and $C + p$
determine different sets of possibilities. Since must $q$ depends exactly on
whether that set of possibilities has only $q$-worlds in it, we get a difference. Thus if $must$ is the consequent of an indicative, context-shiftiness matters.

The trouble for fans of iffiness is that there is interaction between our modal and conditional talk, and no way of sorting out scope issues seems up to getting that interaction right. But that is because we forgot to keep track of the context-shifting job of $if$-clauses. And doing that, even in the simple context-shifting in (20), is enough to make iffiness sit better with the facts.

I know that just one of my marbles is in the box — either Red or Yellow — but do not know which it is. I say we narrowscope the modals. Then all of these are true:

\[(12)\]

a. Modal claim:

(i) Red might be in the box and Yellow might be in the box

(ii) $might \ p \land might \ q$

b. First conditional:

(i) If Yellow isn’t in the box, then Red must be

(ii) $(if \ \neg q) (must \ p)$

c. Second conditional:

(i) If Red isn’t in the box, then Yellow must be

(ii) $(if \ \neg p) (must \ q)$

The facts of the case make sure that $C_i$ has just two worlds in it: $i$, a $(p \land \neg q)$-world and $j$, a $(q \land \neg p)$-world. So (12a) is true at $i$.

So are the two $if$-s. Take (12b). Given the iffiness + shiftiness package, it is true at $i$ in $C$ iff all the possibilities in $C_i \cap \{\neg q\}$ are possibilities that $\langle must \ p \rangle^{C + \neg q}$ maps to true. Thus we have to see whether the following holds:

\[if \ \ k \in C_i \cap \{\neg q\} \ \ then \ \ \langle must \ p \rangle^{C + \neg q,k} = 1\]

If this is so is the $if$ true at $i$ in $C$. But $C_i \cap \{\neg q\} = \{i\}$, so we have to see whether or not $\langle must \ p \rangle^{C + \neg q,i} = 1$. Equivalently: the $if$ is true at $i$ iff $(C + \neg q)_i \subseteq \{p\}$. And since $i$ is in fact a $p$-world the $if$ is true at $i$ in $C$. And mutatis mutandis for (12c).

It is also easy to predict the pattern of entailments in Facts 2 and 3. Narrowscoping the modal in (3):

\[(21)\]

a. For (3a):

(i) If Carl is at the party, then Lenny must be at the party

(ii) $(if \ p) (must \ q)$
b. For (3b):
   (i) If Carl is at the party, then Lenny is at the party
   (ii) \((\text{if } p)(q)\)

Since \(\text{must } q\) entails \(q\), the entailment from (21a) to (21b) is straightforward. So suppose (21b) is true at \(i\) (with respect to \(C\)). Then all of the \(p\)-worlds in \(C_i\) are \(q\)-worlds. But if they are all worlds at which \(q\) is true, then \(i\) — and so, given well-behavedness, every world in \(C_i\) — is equally a world at which \(\text{must } q\) is true. And so (21a) is true, at \(i\) in \(C\), if (21b) is.

And for the existential modal in (6):

(22) a. For (6a):
   (i) If my team wins out, they might win it all
   (ii) \((\text{if } p)(\text{might } q)\)

b. For (6b):
   (i) It might turn out that my team wins out and wins it all
   (ii) \(\text{might } (p \land q)\)

The only noteworthy bit is going from (22a) to (22b). Note that (22a) is true at \(i\) (with respect to \(C\)) just in case all of the \(p\)-worlds in \(C_i\) are worlds where \(\text{might } q\), evaluated in \(C + p\), is true. Well-behavedness guarantees that

\[
\text{if } j, k \in C_i \cap \llbracket p \rrbracket \text{ then } (C + p)_j = (C + p)_k = C_i \cap \llbracket p \rrbracket
\]

If there is a \(q\)-world in \((C + p)_j\), then \(\text{might } q\) is true throughout this set. Since \(\text{might } q\) is an existential modal, if it is true with respect to \(C + p\) it must also be true with respect to \(C\). (Updating contexts with + is monotone.) Whence it follows that the if with a commingling \(\text{might}\) is true at \(i\) iff among the \(p\)-worlds in \(C_i\) lies a \(q\)-world. And any such \(q\)-world will do to witness the truth of \(\text{might } (p \land q)\) at \(i\) in \(C\).

Indicatives play well with epistemic modals. That interaction seemed hard to square with old school iffiness. For, assuming iffiness, the if in \(\text{if } p, \text{ then (modal) } q\) pretty much means \(\text{all}\), saying that the (relevant) worlds where \(p\) is true are all worlds where \(q\) is true. And that’s got to be so whether \(q\) carries a modal decoration or not. But no way of resolving the relative scopes can then be made to fit the facts about how indicatives and modals interact.

But that is because we mistakenly thought that if-clauses only have one job to do. Once we let them do both their index-shifting and context-shifting jobs there is no special problem posed for old school iffiness. It is still true
that *if* pretty much means *all*. Narrowscope the modals. Then *(if p)(might q)*
(in C) means that all the (relevant) *p*-worlds are worlds where *might q* is true.
Similarly: *(if p)(must q)* (in C) means that all the (relevant) *p*-worlds are worlds
where *must q* is true. But the context for figuring out whether, at a given *p*
world, *might q* or *must q* is true is not C but the subordinate or derived context
got by adding the information that *p* to it. This shiftiness makes it plain and
easy to make that fit the facts about how indicatives and modals interact.

Not every fan of old school iffiness will want to follow me this far. But there
is a cost to cutting their trip short since they must then deny or explain away
one of the facts. Iffiness, they’ll no doubt point out, is not without its own costs:
the price of iffiness is shiftiness twice over. That, however, is a bargain.

8 **Bonus material**

If the playground on which you learned of the Ramsey test wasn’t like the one
on which I learned of it then this price might seem high. So I want to end by
pointing out some things that come with it *gratis*.

Whether *(if p)(q)* is true at *i* in C depends both on C and on facts that
obtain at the ever so slightly shifted context C + *p*. That makes the semantics
dynamic in the sense that interpretation both affects and is affected by the
values of contextually filled parameters.

It is also dynamic in the sense that it makes certain sentences “unsta-
bile”—the truth-value a sentence gets in C is not a stable or persistent property
since it can have a different truth-value in a context C’ that contains properly
more information.

(23) **PERSISTENCE**

a. *p* is *t*-persistent iff \([p]^{C,i}_t = 1 \text{ and } C' \subseteq C \implies [p]^{C',i}_t = 1\)
b. *p* is *f*-persistent iff \([p]^{C,i}_f = 0 \text{ and } C' \subseteq C \implies [p]^{C',i}_f = 0\)
c. *p* is persistent iff it is both *f*- and *t*-persistent

The boolean bits are, of course, persistent full-stop. But not the modals: *might*,
being existential, is *f*- but not *t*-persistent; *must* goes the other way. And since
*if* is a strict conditional, equivalent to a necessity modal scoped over a material
conditional, its pattern of persistence is just like that for *must*.

These two senses in which the story is dynamic are two sides of the same
coin. Together they explain how it is that the narrowscoped conditionals
(if $\neg p$) (must $q$) and (if $\neg q$) (must $p$) are consistent with the modalized conjunction might $p \land$ might $q$. From the fact that $i \in [(if \neg p)(must q)]^C$ and $i \in [\neg p]^C$ it does not follow that $i \in [must q]^C$. Indeed, with my marbles lost, this is sure to be false at $i$ in $C$ since might $p$ is true. What is true at $i$ is that — in the subordinate or derived context $C + \neg q \neg must q$ is true. That is allowed because must isn’t f-persistent. But that is not at odds with the might claim. And mutatis mutandis for the other if.

Lack of persistence plus the global behavior of the modals and if’s in the doubly shifty story also make it equivalent to a dynamic story of the indicative that dispenses with the assignment of propositions of the normal sort from the beginning. For suppose we take an information state $s$ to be a set of worlds, and say that what a sentence means is how its LF updates information states. That assigns to sentences the semantic type usually reserved for programs and recipes; they express relations between states — intuitively, the set of pairs between states such that executing the program in the first state terminates in the second. We can think of all sentences in this way thereby treating them as instructions for changing information states. Thus: the meaning of a sentence $p$ is how it changes an arbitrary information state. We might put that by saying the denotation $[p]$ applied to $s$ results in state $s'$; in post-fix notation $s[p] = s'$. Now say that $p$ is true in $s$ iff $s[p] = s$, where $s[p]$ is the result of updating $s$ with $p$.  

Having gone this far, we can make good on the Ramsey test this way:

\[(24) \text{ DYNAMIC IFFINESS} \]

\[s((if \ p) (q)) = \{i \in s : q \text{ is true in } s[p]\}\]

Some programs have as their main point to make such-and-such the case; others to see whether such-and-such. Programs of the latter type are tests and they either return their input state (if such-and-such) or fail (otherwise). That is the


18For the fragment without if’s the updates are as you would expect. For the if-free fragment of $L$, define $[\cdot]$ as follows:

- $s[p_{atomic}] = \{i \in s : i(p_{atomic}) = 1\}$
- $s[\neg p] = s \setminus s[p]$  
- $s[p \land q] = s[p]\{q\}$  
- $s[might \ p] = \{i \in s : s[p] \neq \emptyset\}$

It then follows straightaway that — for the if-free fragment — $s[p] = s \cap [p]$.  

19This generalizes the plain vanilla story about satisfaction we were taught when first learning propositional logic: as the story usually goes, a boolean $p$ is true relative to a set of possibilities $s$ iff all the possibilities in $s$ are in $[p]$. But that is equivalent to saying that adding $[p]$ to the information in $s$ produces no change: $s \cap [p] = s$ iff $s \subseteq [p]$.  


kind of program (24) says \textit{if} is. In good Ramseyian spirit, an \textit{if} tests \(s\) to see whether the consequent is true in the subordinate context got by hypothetically adding \(p\) to \(s\). Truth isn’t persistent here, either. That is because a state may pass a test posed by an existential (are there \(p\)-possibilities?) and have some narrower, less uncertain state fail it (no more \(p\)-possibilities). And dually for the universal \textit{must} and \textit{if}.

This is what makes the story non-trivially dynamic, and what mimicks in this set-up what the doubly shifty story does.\(^{20}\) So even though I told the story about truth-values assigned at contexts and indices, it is equivalent to a story about changing information states.

I put that in the “bonus” column, but not everyone is similarly inclined. Still, there is a bonus to be had. There are natural properties that indicative conditionals seem to have but which — at least so I was taught — no propositional operator can have all at once. Suppose \textit{if} is at least as strong as material implication, no stronger than strict implication, and treats (25a) and (25b) as true in the same spots.

\begin{enumerate}[\textbf{(25)}]
\item If Carl is away, then if Lenny is away then Sector \(7G\) is empty
\item If Carl is away and Lenny is away, then Sector \(7G\) is empty.
\end{enumerate}

Then — again, so I was taught — \textit{if} must have the truth-conditions of material implication. Riots in the streets, dogs and cats living together, mass hysteria, and all the rest.\(^{21}\)

Some have gotten used to this sad state of affairs either by dropping a condition — the equivalence in (25) for fans of variably strictness — or embracing the horseshoe. And some have marshaled anti-iffiness to save our bacon.\(^{22}\) Better to find a way out than live with those choices.

And iffiness — either in the doubly shifty version or the full-on dynamic version — buys you exactly that. That is a bonus — but not exactly \textit{gratis} since saying exactly how seems like a different paper.\(^{23}\)

\(^{20}\)The standard benchmark for \textit{bona fide} dynamics is whether the interpretation function \([\cdot]\) is either \textit{non-introspective} (Can it be that \(s[p] \not\in s?)\) or \textit{non-continuous} (Can it be that \(s[p] \not\in \bigcup_{i \in s} \{i[p]\}\)). In this set-up, the behavior of indicatives is not continuous.

\(^{21}\)For versions of the argument see Gibbard (1981); Veltman (1985); Kratzer (1986); Edgington (1995, 2006); Gillies (2007b).

\(^{22}\)In particular, Kratzer (1986).

\(^{23}\)And is: see Gillies (2007b).
Bibliography


von Fintel, K. and S. Iatridou (2003). “*If* and when *if*-clauses can restrict quantifiers,” ms. MIT.


