The Temperature Paradox as Evidence for a Presuppositional Analysis of Definite Descriptions

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Montague’s (1973) analysis of the temperature paradox is well-known. Sentences (1), (2) and (3) are assigned logical translations equivalent to (4), (5) and (6), respectively.

(1) The temperature rises.
(2) The temperature is ninety.
(3) Ninety rises.

(4) \( \exists x [\forall y [\text{temperature}'(y) \leftrightarrow x = y] \land \text{rise}'(x)] \)
(5) \( \exists x [\forall y [\text{temperature}'(y) \leftrightarrow x = y] \land 'x = n] \)
(6) \( \text{rise}'('n) \)

Here, \( x \) and \( y \) are variables of type \( \langle s, e \rangle \), ranging over “individual concepts” (functions from indices to individuals); \( \text{temperature}' \) and \( \text{rise}' \) are predicates of type \( \langle \langle s, e \rangle, i \rangle \), taking individual concepts as arguments; \( n \) is a constant of type \( e \), denoting an individual (presumably the number 90); \( 'x \) is an expression of type \( e \), denoting the individual yielded by \( x \) for the index of evaluation; and \( 'n \) is an expression of type \( \langle s, e \rangle \), denoting the constant function which yields at every index the individual denoted by \( n \). In this way, sentence (1) is analyzed as asserting that the unique temperature function rises; (2) is analyzed as asserting that the value of this function for the index of evaluation is 90; and (3) is analyzed as asserting that the function which picks out 90 at all indices rises. Because the temperature function might yield 90 at the index of evaluation, without being identical to the function which yields 90 at all indices rises. Because the temperature function might yield 90 at the index of evaluation, without being identical to the function which yields 90 at all indices rises, sentences (1) and (2) may be true even while (3) is false, resolving the paradox.

This analysis has been criticized on a number of grounds. In some cases, however, it is possible to reconstruct the temperature paradox using examples to which these criticisms do not apply.

For example, Jackendoff (1979) argues that sentence (2) should not be analyzed as equative, and that the paradox disappears if we treat it instead as locative, on analogy to sentences like (7).

(7) The temperature is at ninety.
But as Thomason (1979) hints and Löbner (1981) points out explicitly, it is possible to reconstruct the paradox using examples which seem more clearly equative rather than locative, e.g. (8), (9) and (10):

(8) The temperature in Chicago is rising.
(9) The temperature in Chicago is the very same as the temperature in St. Louis.
(10) The temperature in St. Louis is rising.

So even if we accept Jackendoff’s criticism for examples like (1) through (3), something like Montague’s analysis might still be appropriate for examples like (8) through (10).

A different line of criticism comes from Bennett (1975), who expresses reservations about treating numbers as individuals, and also complains that Montague’s analysis does not appeal at all to the notion of measurement, which seems intuitively involved in the temperature sentences. He suggests that temperatures might be better represented as functions from indices to “entities such as twenty degrees Fahrenheit.” This sort of complaint is supported by the fact that sentences like (2) may be paraphrased as in (11):

(11) The temperature is ninety degrees Fahrenheit.

Similar concerns provide part of the motivation for the analysis of Thomason (1979).

But nothing in Montague’s original analysis actually required us to regard temperatures as functions from indices to numbers; all that is formally required is that they be functions from indices to individuals of some kind or other. The presumption that these individuals must be numbers comes only from the assumption that the name ninety must denote a number. There is actually nothing to prevent us from regarding ninety in (2) and (3) as elliptical for ninety degrees Fahrenheit, and understanding the temperature function in Montague’s analysis as picking out a “degree” at each index rather than a number; nothing in the formalism need change.²

Moreover, sentence (2) may also be paraphrased as in (12), and in this case it seems quite reasonable to treat the subject noun phrase as denoting a number.

(12) The temperature in degrees Fahrenheit is ninety.

We could reconstruct the entire paradox using this example instead of (2).³

So none of these objections seem to me to seriously undermine Montague’s basic strategy in resolving the paradox.

A more significant problem, in my view, is pointed out by Dowty, Wall and Peters (1981: 284-285), who credit Anil Gupta with discovering it. In Montague’s analysis, nouns like temperature
and \textit{price} are treated as predicates applying to individual concepts. That is, each of them denotes a function of type \(\langle s, e, t \rangle\) — in effect, a set of individual concepts. Like any non-logical expression, a noun may receive a potentially different denotation at each index; so \textit{price}, for example, may denote a different set of individual concepts at each index, as may \textit{temperature}. This gives the result that certain intuitively valid arguments are predicted to be invalid.

To see this, consider a model containing just three indices\(^4\): \(i_1\), \(i_2\) and \(i_3\). Relative to \(i_1\), let \textit{temperature} denote the singleton set containing the individual concept T1, and let \textit{price} denote the singleton set containing the individual concept P1. Likewise, relative to \(i_2\), let \textit{temperature} denote \{T2\} and \textit{price} denote \{P2\}, and relative to \(i_3\), let \textit{temperature} denote \{T3\} and \textit{price} denote \{P3\}.

Now suppose T1–T3 and P1–P3 are as given in (13) and (14):

\begin{align*}
(13) & \quad T1(i1) = 99 \quad T2(i1) = 89 \quad T3(i1) = 79 \\
& \quad T1(i2) = 100 \quad T2(i2) = 90 \quad T3(i2) = 80 \\
& \quad T1(i3) = 101 \quad T2(i3) = 91 \quad T3(i3) = 81 \\
(14) & \quad P1(i1) = 99 \quad P2(i1) = 91 \quad P3(i1) = 83 \\
& \quad P1(i2) = 98 \quad P2(i2) = 90 \quad P3(i2) = 82 \\
& \quad P1(i3) = 97 \quad P2(i3) = 89 \quad P3(i3) = 81
\end{align*}

Let us interpret the verb \textit{rise} in the intuitive way, assuming that \(i_1\) precedes \(i_2\) and \(i_2\) precedes \(i_3\): at \(i_2\), for example, we let \textit{rise} hold of an individual concept \(F\) iff \(F(i1) < F(i2) < F(i3)\).

Now consider the argument with (15) and (16) as premises and (17) as its conclusion:

\begin{align*}
(15) & \quad \text{Necessarily, the temperature is the price.} \\
(16) & \quad \text{The temperature rises.} \\
(17) & \quad \text{The price rises.}
\end{align*}

These sentences are assigned logical translations equivalent to (18) through (20), respectively:

\begin{align*}
(18) & \quad \square \exists x[\forall y[\text{temperature}'(y) \cdot x = y] \land \exists z[\forall w[\text{price}'(w) \cdot z = w] \land \forall x = z]] \\
(19) & \quad \exists x[\forall y[\text{temperature}'(y) \cdot x = y] \land \text{rise}'(x)] \\
(20) & \quad \exists x[\forall y[\text{price}'(y) \cdot x = y] \land \text{rise}'(x)]
\end{align*}

Intuitively, the argument seems valid, but it is easy to see that (18) and (19) can be true while (20) is false. At \(i_2\) in the model sketched above, for example, (18) is true, because for every index \(i\), the unique temperature function relative to \(i\) and the unique price function relative to \(i\) yield the same value for \(i\): At \(i_1\), T1 and P1 both yield the value 99; at \(i_2\), T2 and P2 both yield the value 90; and at \(i_3\), T3 and P3 both yield the value 81. Likewise, (19) is true, because the unique temperature function relative to \(i_2\) is T2, and T2(i1) < T2(i2) < T2(i3) (that is, 89 < 90 < 91).
Yet (20) is false (relative to \(i2\)), because the unique price function relative to \(i2\) is \(P2\), and \(P2(i1) \lessdot P2(i2) \lessdot P2(i3)\) (that is, \(91 \lessdot 90 \lessdot 89\)).

As Dowty, Wall and Peters imply in their discussion, this problem could be easily fixed by adding suitable meaning postulates. But such an approach would be stipulative at best; one cannot help but wonder if there might be a more “architectural” solution to the problem.

In fact I think there is a preferable solution to the problem which becomes obvious once we consider what the motivation is for analyzing nouns like temperature and price as predicates of individual concepts. In considering this issue, we should be careful to distinguish the motivation for treating rise in this way from the motivation for treating temperature and price this way, because reasons are quite different in the two cases:

*Rise* is treated as taking individual concepts as its arguments because it is impossible to determine whether a function \(f\) is rising at index \(i\) simply by examining the value which \(f\) yields for \(i\). Instead, we must also know what values \(f\) yields for neighboring indices, to see if the earlier ones yield lower values and the later one yield higher values. Put somewhat differently, you cannot determine whether the temperature is rising at a given moment by examining a snapshot of a thermometer taken at that moment — you need more than one snapshot, taken at different times. This sort of consideration makes it seem quite reasonable to regard rise as creating an authentic temporally intensional context, and to analyze it as taking individual concepts, rather than individuals, as its arguments.

For temperature and price the situation is quite different. To know that a particular value is the temperature at a given moment, a single snapshot suffices, and snapshots taken at other times are essentially irrelevant. Nor do you need to know anything about the price history of a product to know its current price. Temperature and price do not intuitively require consideration of multiple indices to determine their extensions, and taken purely on their own ground, they do not provide any reason for an analysis in which they take individual concepts rather than individuals as their arguments.

Why, then, did Montague give an analysis in which their arguments were individual concepts? Only because of this: In formula (4), repeated here as (21), the variable \(y\), with which the predicate temperature’ combines, must be of the same type as the variable \(x\), with which rise’ combines; otherwise, the clause ‘\(x = y\)’ will not be well-formed, or make any sense. But the variable with which rise’ combines must be of type \(\langle s, e \rangle\), and so the variable with which temperature’ combines will have to be of type \(\langle s, e \rangle\) as well.

\[
\exists x[\forall y[\text{temperature}'(y) \dashv x = y] \land \text{rise}'(x)]
\]

Why do we need these variables? Only because Montague assumes a Russelian analysis of the definite determiner, in which it expresses unique existential quantification. If we were to adopt a presuppositional analysis of definites, the need for the variables disappears, and with it the
requirement that the variables must match in type. This removes the motivation for treating temperature and price as being of type \(\langle s, e, t \rangle\), and allows an analysis in which the argument in (15) through (17) comes out valid without the extra stipulation of meaning postulates.

To see this, add to the syntax and semantics of Montague’s IL the following rules:

(22) If \(\phi \in \text{ME}_a\) and \(u\) is a variable of type \(a\), then \(u\phi \in \text{ME}_a\)
(23) \([u\phi]^{\mathfrak{A},i,j,g}\) is the unique object \(d \in D_{a,A,I,J}\) such that \([\phi]^{\mathfrak{A},i,j,g'} = 1\) (where \(g'\) is the \(\mathfrak{A}\)-assignment like \(g\) except for the possible difference that \(g'(u) = d\)) if such an object exists; undefined otherwise.

These rules sometimes produce expressions with an undefined semantic value, and this effect will propagate up the syntactic tree, yielding a truth value gap. We take such cases to be examples of presupposition failure: an expression of the form \(\lambda u\phi\) carries a presupposition that there is a unique object satisfying \(\phi\). Readers who wish to limit upward propagation of the truth value gap are invited to adapt the present rules to their favorite system of three-valued logic or supervaluations.

We now stipulate that common nouns receive translations of type \(\langle e, t \rangle\) rather than \(\langle (s, e), t \rangle\).

Next, revise the rule for translating definite noun phrases into Intensional Logic (given as part of T2 in Montague 1973) as follows, where \(u\) should be understood as a variable of type \(e\), and \(P\) as a variable of type \(\langle s, \langle (s, e), t \rangle \rangle\):

(24) If \(\zeta \in P_{\text{CN}}\) and \(\zeta\) translates as \(\zeta'\), then the \(\zeta\) translates as \(\lambda P\{\lambda u\zeta'(u)\}\)

With these rules in place, sentences (15) through (17) receive translations equivalent to (25) through (27), respectively:

(25) \(\Box u\text{temperature}'(u) = u\text{price}'(u)\)
(26) \(\text{rise}'(\lambda u\text{temperature}'(u))\)
(27) \(\text{rise}'(\lambda u\text{price}'(u))\)

Note that \(\text{rise}'\) continues to take a type \(\langle s, e \rangle\) argument, just as before. But instead of a variable ranging over the full set of type \(\langle s, e \rangle\) functions, the argument in this case is formed by prefixing the intensional operator \(^\wedge\) to the type \(e\) expression \(u\text{temperature}'(u)\). This operator is defined in the standard way:

(28) \([\alpha]^{\mathfrak{A},i,j,g} is that function \(f\) with domain \(I \times J\) such that for all \(i' \in I, j' \in J: f(i', j') = [\alpha]^{\mathfrak{A},i',j,g}\)

Note that this definition guarantees that \(^\wedge\alpha\) will always denote the same function, regardless of choice of indices of evaluation; \([\alpha]^{\mathfrak{A},i,j,g} = [\alpha]^{\mathfrak{A},i',j,g}\) for all \(i, i', j, j'\).
In particular, ‘\(u\)temperature\(u\)’ will always denote the same function, which we may regard as the function which picks out the temperature at each index. Likewise ‘\(u\)price\(u\)’ will always denote the same function, picking out the price at each index. Since neither of these expressions varies from index to index in which function it picks out, we cannot obtain a situation like that presented in (13) and (14), with a different temperature function at each index and a different price function at each index; instead we have a single temperature function for all indices and a single price function for all indices.

Now suppose (25) is true. Then ‘\(u\)temperature\(u\)’ = ‘\(u\)price\(u\)’ is true at all indices; that is, the denotation of ‘\(u\)temperature\(u\)’ at any given index is identical to the denotation of ‘\(u\)price\(u\)’ at that index. Therefore, the function which picks out the denotation of ‘\(u\)temperature\(u\)’ at any given index must be identical to the function which picks out the denotation of ‘\(u\)price\(u\)’ at any given index. In other words ‘\(u\)temperature\(u\)’ must denote the same function as ‘\(u\)price\(u\)’.

But then, if (26) is true, (27) must be true as well. That is, the argument from (25) and (26) to (27) is valid. Since these formulas are the translations of (15), (16) and (17), the argument from (15) and (16) to (17) is valid as well.

We thus obtain intuitively correct results for these examples, without the extra stipulation of meaning postulates for ‘price’ and ‘temperature’. These results are made possible because we are treating common nouns, including ‘price’ and ‘temperature’, at their intuitively correct type of ‘\(e, t\)’, instead of the higher type ‘\(s, e, t\)’ proposed by Montague. The use of this higher type was necessitated by Montague’s assumption of a Russellian, quantificational analysis of definite noun phrases (combined with the treatment of intensional verbs like ‘\(rise\)’ as being of type ‘\(s, e, t\)’); once we drop the Russellian analysis in favor of a presuppositional analysis, the simpler type assignment becomes possible, and the validity of the argument in (15) to (17) falls out automatically, eliminating a significant problem in Montague’s treatment of the temperature paradox. Because the presuppositional analysis makes possible an improved analysis of the temperature paradox, we may regard the temperature paradox as providing evidence in favor of the presuppositional account of definites.

Endnotes

1. The function is guaranteed to be constant by a meaning postulate, not shown here.

2. Bennett would prefer not to treat degrees as primitive individuals, and suggests instead treating them as functions from indices to sets of individuals of equal temperature; so this move would not quite satisfy his complaint. But regardless of how we represent degrees, the general strategy for resolving the paradox seems much the same: verbs like ‘\(rise\)’ apply to functions from indices to degrees, while ‘\(be\)’ asserts that two functions yield identical values at a particular index, not that the
functions themselves are identical.

3. This would involve replacing (1) with (a), which is rather unnatural-sounding — but only, it seems to me, because if the temperature is rising, it is rising no matter whether measured in degrees Fahrenheit, Celsius, Kelvin, or whatever; the prepositional phrase is redundant, hence pragmatically infelicitous. This should not affect the logical status of the sentence, or its role in the paradox:

(a) The temperature in degrees Fahrenheit rises.

4. Montague’s original analysis relativized interpretation to *pairs* of indices, one index from each pair representing a possible world, and the other index representing a time. Here, I simplify slightly by dropping the world index.

5. Postulates of the form $\forall x[\alpha(x) \rightarrow \Box \alpha(x)]$, where $\alpha = \text{temperature}' or 'price' would do the trick. Note that $x$ ranges over functions from indices to individuals; this formula does *not* mean that if the temperature is 90 at the index of evaluation, it must be 90 at all indices.

6. We revert here to Montague’s original system, in which interpretation is relativized to pairs of indices. I depart slightly from Montague’s notation in using the now-standard double brackets to indicate semantic values.

**References**


Thomason, Richmond H. (1979) ‘Home is Where the Heart Is’. *Contemporary Perspectives in
the Philosophy of Language, 209-219, Peter A. French, Theodore E. Uehling, Jr., and Howard K. Wettstein, eds. Minneapolis: University of Minnesota Press.