Domain restriction by conditional connectives

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Abstract
This paper aims to rehabilitate the idea that there are conditional connectives in the logical forms of natural language. I propose to adopt a connective with a truth-value gap semantics inspired by Belnap (1970), such that conditionals only have a truth-value if their antecedent is true. Together with the assumption that quantifiers select worlds for which their scope is defined, this predicts that if-clauses under quantifiers serve to restrict the domain of quantification. The proposed semantics allows for a straightforwardly compositional analysis of quantified conditionals.

1 The decline of the conditional connective

1.1 Introduction
The present paper concerns the interpretation of if-clauses in the scope of adverbial and modal quantifiers as in ‘Harry usually drinks Butterbeer if
he is happy’. Such if-clauses intuitively restrict the domain of the higher quantifier. Linguistic folklore has it that we can only account for this by denying that if corresponds to a conditional connective. The main argument goes back to Lewis (1975) who concluded that embedded if-clauses just have no meaning apart from the quantifier whose domain they restrict. In various contributions, Kratzer (1978, 1979, 1981, 1986, 1991) generalized this idea to all if-clauses, including so-called ‘bare’ conditionals that do not contain any overt quantifier. This analysis of conditionals, known as the Lewis/Kratzer analysis, has become the standard analysis of conditionals, at least of those of the domain-restricting kind (those embedded under modal quantifiers).

However, it seems that in some cases we do want to say that if corresponds to a conditional connective. We need non-modalized conditional propositions to serve as antecedents for sentence anaphors. As a bonus, this helps to account for so-called Gibbardian standoffs. This all suggests that we need a new analysis of domain restriction by if-clauses, one that is compatible with there being conditional connectives in the logical forms of natural language. The present paper proposes such an analysis. Following Belnap (1970, 1973) I will propose that if corresponds to a connective with a partial semantics. The idea is that conditionals only have a truth value in case their antecedent is true, and that their truth value in that case equals the truth value of the consequent. Together with the assumption that quantifiers only select worlds in which their scope is defined, this predicts that if-clauses in the scope of a quantificational operator restrict the domain of that operator.

This paper is structured as follows. In the remainder of this section, I will reprise the history of the decline of the conditional connective, starting
with Lewis’ (1975) seminal paper. Then, in section 2, I will point out two problems for the Lewis/Kratzer approach to conditionals. In section 3, I will propose a truth-value gap semantics which solves these problems. Finally, in section 4, I will examine the logical consequences of this partial semantics. I will argue that, although we lose the validity of certain logical laws by moving to a non-classical system, this is not nearly as problematic as one would think.

1.2 Background assumptions

Before we delve into the details of Lewis’ argument, I would like to introduce some background assumptions. First, I assume that adverbs like *always*, *usually*, and *seldom* quantify over events. This is not Lewis’ view, but nothing hinges on that. Lewis did hold that the range of quantification for these adverbs is often restricted. Sentence (1), for instance, is not true just because few of all events are events in which Caesar was even alive, from which it follows that fewer still are events in which he awoke before dawn. Rather, (1) means something like the following: few events in which Caesar awoke are events in which he awoke before dawn:

(1) Caesar seldom awoke before dawn.

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1Lewis took these adverbs to be unselective binders, i.e. operators that bind every free variable in their scope. This idea, especially as developed by Kamp (1981) and Heim (1982), has been very successful in accounting for quantificational variability effects and for donkey anaphora. For the purposes of this paper, however, these issues are of no concern.
In general, domain selection is influenced by information structure: backgrounded material (whether topical or presupposed) is typically understood as part of the quantifier’s restrictor (see von Fintel (1994, ch.2) for an overview). As for (1), it seems that this sentence is likely to be interpreted with *before dawn* in focus, and *awoke* as part of the topic; hence quantification ranges over events in which Caesar awoke.

To represent restricted quantification, I will make use of logical forms that combine quantifiers both with a restrictor and a nuclear scope. The restrictor determines the domain of the quantifier, while the nuclear scope constitutes the main predication that is made about the (relevant quantity) of members of the domain. Let us represent the meaning of (1) as follows:

\[
(\text{few } e: \text{Caesar awoke in } e)(\text{Caesar awoke before dawn in } e)
\]

Logical forms of this type go back to McCawley (1981), and are adopted here mainly because they are easy to read. (2) is true in a world \(w\) iff few values of \(e\) that satisfy the restrictor in \(w\) also satisfy the nuclear scope in \(w\), i.e. iff few events in \(w\) in which Caesar awoke are events in which he awoke before dawn.

### 1.3 The argument

We now have everything in place to consider Lewis’ reasons for claiming that (some) *if*-clauses do not correspond to conditional connectives. The most prominent candidate is the material implication \(\supset\), but it is easy to see that this connective is not up to the task. Universal adverbs, like *always*, are
not yet problematic: conditionals in their scope can easily be interpreted in terms of $\supset$, as witnessed by the following representation for (3). Note that I abstract away from any contextual restrictions on the domain, i.e. the restrictor is empty. We will see below that this doesn’t affect the force of Lewis’ argument.

(3) Harry always drinks Butterbeer if he is happy.

$$(\text{all } e)(\text{Harry is happy in } e \supset \text{he drinks Butterbeer in } e)$$

This logical form is true in a world $w$ iff all values of $e$ that satisfy the restrictor in $w$ also satisfy the nuclear scope in $w$, i.e. iff for each event in $w$: either it isn’t an event in which Harry is happy, or it is an event in which he drinks Butterbeer. In effect, then, (3) is true iff each event in $w$ in which Harry is happy is an event in which he drinks Butterbeer.

But under *usually*, the same analysis of conditionals fails. Consider (4) and its logical representation:

(4) Harry usually drinks Butterbeer if he is happy.

$$(\text{most } e)(\text{Harry is happy in } e \supset \text{he drinks Butterbeer in } e)$$

The representation in (4) is too weak: it is satisfied iff Harry is usually unhappy. To see this, suppose that there are 1000 events in $w$, and that Harry is happy in only 300 of them. Of these 300 events, 100 involve Harry drinking Butterbeer. Intuitively, (4) is false, but the analysis predicts it to be true. Dekker (2001, 7), one of the advocates of material implication, is not too worried about this problem. He acknowledges that material implication
makes the wrong predictions, but writes: “I leave it to the general public to judge the issue”. For me, however, the logical form above is simply not a good enough approximation of the meaning of (4), nor was it for Lewis, and I believe that the general public agrees. Lewis concluded:

the if of our restrictive if-clauses should not be regarded as a sentential connective. It has no meaning from the adverb it restricts. [...] It serves merely to mark an argument-place in a polyadic construction. (Lewis 1975, 11)

This leads to the following improved logical form for (4), in which the if-clause is part of the adverb’s restrictor:

(5) (most e: Harry is happy in e)(Harry drinks Butterbeer in e)

This is true in a world w iff most events of w in which Harry is happy are events in which he drinks Butterbeer.

Regardless of how representations like (5) relate to the grammar of English, it is clear that on Lewis’ theory, the if in sentences like (4) doesn’t contribute its usual conditional meaning. In fact, logical forms like (5) do not contain anything that corresponds to if, but opinions diverge on whether this means that if is semantically vacuous. Some compositional implementations of Lewis’ proposal, e.g. von Stechow (2004), do make this assumption, but others, e.g. von Fintel (1994), still endow the element with some meaning. Yet, crucially, this meaning is far from reminiscent of the meanings of familiar conditional connectives.
It should be noted that the representation in (4) cannot easily be saved by appealing to appropriate contextual restrictions on the domain. It is true that if the context somehow supplied a restrictor that matched the *if*-clause, the observed interpretation would be predicted:

\[(6) \quad (\text{most } e: \text{Harry is happy in } e) (\text{he is happy in } e \supset \text{he drinks B-beer in } e)\]

This representation has the same truth conditions as (5). But this analysis can only succeed if we can come up with a good story about why and how the context would supply exactly this restriction, and this turns out to be quite difficult. My best bet is the following. *If*-clauses introduce topics, cf. Haiman (1978), and domain selection is sensitive to topical information. We have already noted this in connection to (1), but further prima facie evidence for the latter claim comes from the following example by Beaver (2004), who traces back the leading idea to Partee (1991); see also Eckardt (1999):

\[(7) \quad \text{Whatever other options are available, it is by \text{PUBlic TRANsport} that most \text{BRItish} go to work. In contrast, all \text{iTALians} use their \text{CARS} to go to work.}\]

It is natural to interpret the first sentence as meaning that most British *who go to work* do so by public transport, and the second as meaning that all Italians *who go to work* do so by car. Beaver argues that the stress-pattern leads the hearer to infer that going-to-work is the topic of the discourse, and that this licenses domain restriction with the set of work-goers.
Could it be that in our sentence (4) the domain of the adverb is restricted in a similar indirect fashion? In that case, the if-clause signals that Harry’s habits when he is happy are under discussion in the present discourse. When evaluating the restrictor of the quantifier, people take into account what the discourse is about, and thus the domain of usually gets restricted to events in which Harry is happy. This sounds promising, but it is ultimately not tenable. As pointed out by von Fintel (1994, 82), if-clauses may be foregrounded:

(8) A: Are you going to play soccer on Sunday?
   B: We’ll play [if the SUN shines]f.

B communicates that we will play soccer in all events in which the sun shines. Yet the if-clause is in focus. Crucially, it is not the topic. Now note that if-clauses in the scope of adverbial quantifiers need not be topical either:

(9) A: When do you play soccer?
   B: We usually play [if the SUN shines]f

Summing up, some if-clauses are used as devices for restricting the domain of some higher operator. This is difficult to account for if if corresponds to a classical conditional connective, so Lewis proposed that some if-clauses are mere domain restrictors. Lewis did not intend this analysis to be applied across the board, but it is only a small step to generalize the main idea to all conditionals.² Kratzer (1978, 1979, 1981, 1986, 1991) argues that if there is no overt quantifier present in a conditional, we must assume that a covert

²In Lewis (1976) he championed material implication as the best analysis for ordinary (non-quantified) indicative conditionals.
universal quantifier is present:

The history of the conditional is the story of a syntactic mistake. There is no two-place if...then connective in the logical forms of natural languages. If-clauses are devices for restricting the domains of various operators. Whenever there is no explicit operator, we have to posit one. (Kratzer 1991, 656)

The covert operator is usually an epistemic necessity modal. Consider for instance (10a), and note that this sentence is more or less equivalent to (10b):

(10) a. If Harry is not in the library, he is in the common room.
    b. If Harry is not in the library, he must be in the common room.

In Kratzer’s analysis, both sentences receive the following logical form:

(11) (all w: w is compatible with the evidence and Harry is not in the library in w)(he is in the common room in w)

Thus, the domain of the modal is contextually restricted to epistemic alternatives, and the if-clauses in (10) supply an additional restriction on this domain. (11) is true in a world w iff Harry is in the common room in values for w that are compatible with the available evidence in w and in which Harry is not in the library.³

³It has been argued that if-clauses may also be used to restrict the domains of nominal quantifiers, see for instance von Fintel (1998). But von Fintel and Iatridou (2002) and Higginbotham (2003) argue that sentences like ‘No student will succeed if he goofs off’ do not mean the same as ‘No student who goofs off will succeed’. See Huitink (2009b) for an overview of the recent literature on this issue.
1.4 Compositional implementation

Within the Lewis/Kratzer approach to conditionals little is said about the compositional interpretation of conditionals. To the best of my knowledge, von Stechow (2004) and von Fintel (1994) are the only exceptions. One of the issues to address is that the Lewis/Kratzer approach presupposes a syntax-semantics mismatch: at surface, *if*-clauses occur in a sentence-initial position, but semantically, they are the argument of some higher quantifier. The null-hypothesis which accounts for this is that in a conditional like ‘If it is snowing, it must be cold’ the subordinate clause is base-generated as a sister to the quantifier’s restrictor variable $\rho$, in which position it is also interpreted, while overt word order is derived by moving the *if*-clause. This is the proposal by von Stechow (2004). Accordingly, the LF for our sentence is as in (12) (as is standard, I assume that the expletive subject is also lowered to the position where it was base-generated):

(12)  \[\text{must} \ [\rho] \ [\text{if it is snowing}] [\text{it be cold}]\]

The denotation of the entire *if*-clause is just the proposition that it is snowing (*if* is semantically vacuous). This proposition serves to restrict the set of worlds that the assignment function supplies for $\rho$ (see Huitink (2008, 180-185) for a more formal presentation).

In von Fintel’s approach, on the other hand, *if*-clauses restrict the domain of the relevant quantifier through co-indexation, without being a syntactic argument of the operator. This is an *in situ* approach, where the *if*-clause binds the free restrictor variable and as such constrains the interpretation of
that variable. This leads to the following LF:

\[(13) \quad [\text{if it is snowing}][[\text{must } \rho] \ [\text{it be cold}]]\]

In von Fintel’s approach, the if-clause does carry some meaning: it tells the variable assignment function to take it into account when supplying a value for \(\rho\). As said above, even though if is not strictly speaking meaningless on this implementation, if clearly does not correspond to a conditional connective. Rather, if is a mere domain restrictor.

2 Rehabilitating the conditional connective

Let’s now pause for a moment and consider whether we really want to give up the assumption that there are conditional connectives in the logical forms of natural language sentences. Close inspection reveals that we do not. First, this does not fit the interpersonal traffic of conditionals very well. Second, it seems that conditional connectives, at least those of the kind that I favor, can help in what I will call, following Stephenson (2007b), Gibbardian standoffs. I will explain these problems in turn.

2.1 Conditionals in dialogue

We want conditional embedded under quantifiers to be the antecedent of sentence anaphors. This argument comes from von Fintel (2007) (in turn,
he credits Brian Weatherson\textsuperscript{4}). Consider the following dialogue:\textsuperscript{5}

\begin{flalign}
(14)\quad & A:\text{ If this cat isn’t male, it is female.} & \\
& B:\text{ Necessarily so.} & \\
& C:\text{ That’s very unlikely.} &
\end{flalign}

The problem is this. Assuming that \textit{so} and \textit{that} are sentence anaphors, they have to refer back to some sentence in \textit{A’s} utterance. But under the Lewis/Kratzer analysis, there is no sentence in \textit{A’s} utterance which could be substituted for \textit{so} or \textit{that} such that the result corresponds to the intuitive meaning of the utterances by \textit{B} and \textit{C}. Let’s focus on \textit{B’s} statement. It seems that this sentence must be interpreted as if \textit{necessarily} embeds a conditional with a restrictive \textit{if}-clause. That is, (assuming that we are working in the Lewis/Kratzer framework), we would like to derive the following logical form:

\begin{flalign}
(15)\quad & (\text{all } w: w \text{ is compatible with the evidence and this cat isn’t male in } w) (\text{this cat is female in } w) &
\end{flalign}

In other words: \textit{B} says that in all epistemic alternatives in which this cat is not male, it is female. Similar remarks apply to \textit{C’s} utterance, i.e. quantifi-

\textsuperscript{4}I have not been able to find a paper by Weatherson where he actually formulates this argument, although Weatherson (2009) does discuss dialogues as:

(i)\quad & A:\text{ If the doctor didn’t do it, the lawyer did.} & \\
& B:\text{ That’s right.} &

However, Weatherson uses such examples to argue in favor of a relativistic analysis of conditionals. I think of the truth-value gap semantics proposed in the present paper as an alternative to relativism (see section 2.2 below).

\textsuperscript{5}This is not von Fintel’s original example. I changed it slightly to bring out the point more clearly. I thank Regine Eckardt (p.c.) for suggesting this change.
cation ranges over worlds which satisfy the if-clause. C thus says here that in hardly any of the alternatives in which this cat isn’t male, it is female (never mind that it follows that C is thus confused about the gender possibilities for cats).

It seems, however, that there is no way to derive these meanings, if if doesn’t correspond to a conditional connective. Given the Lewis/Kratzer analysis, A’s utterance contains two sentences that are candidate referents for so and that. First, these anaphors could refer back to the matrix sentence, which is in this analysis a modal complex. Second, they could refer back to the embedded consequent (assuming that this sentence is accessible to the anaphors). Neither option gives us what we want. To see this, suppose that in B’s utterance, so refers back to the matrix sentence in A’s utterance. This would lead to the following representation:

\[(16) \quad (\text{all } w: w \text{ is compatible with the evidence})((\text{all } v: v \text{ is compatible with the evidence in } w \text{ and this cat isn’t male in } v)(\text{it is female in } v))\]

This expresses a doubly modalized proposition: that it is necessary that it is necessary that this cat is female in those worlds in which it is not male. This is not the reading we want, because we want just one layer of modality as in (15).\(^6\) The sentence anaphors must thus be concluded not to pick up the matrix sentence.

\(^6\)Note that it won’t do to plead modal concord. As argued in Huitink (2009a) modal concord is only possible between modals of equivalent force and flavor. The modal in B’s utterance may be similar to the covert modal in A’s utterance in these two respect, but the modals in C’s utterance are definitely not. Thus, modal concord will not save all the problematic examples.
The alternative is that the anaphors pick up the embedded consequent, i.e. ‘this cat is female’. The problem with this option is that we still need to have the domains of the modals in the statements by B and C restricted to worlds in which this cat is not male. Simply plugging in the consequent leads to (17) (for B):

\[
(17) \quad (\text{all } w: w \text{ is compatible with the evidence})(\text{this cat is female in } w)
\]

Again, this is not the same as (15). In other words, the alternative analysis falsely predicts that B’s statement means that it is necessary *simpliciter* that this cat is female (independent of this cat not being male).

Can we, to save the idea that *so* refers back to the consequent, assume that B’s statement is implicitly conditionalized? As argued by von Fintel (2007), it seems that we cannot. That is, implicit conditionalization is not generally available as an interpretation strategy for modal expressions. For the following dialogue we don’t want to say that the overt modals are restricted to worlds in which this cat isn’t male:

\[
(18) \quad \begin{align*}
A: & \quad \text{If this cat isn’t male, it is female.} \\
B: & \quad \text{This cat is necessarily female.} \\
C: & \quad \text{It’s very unlikely that this cat is female.}
\end{align*}
\]

In *this* dialogue, B’s utterance *does* correspond to the representation in (17) rather than to the one in (15). That is, B is saying that she has evidence to conclude that this cat is female. Similar for C: she is saying that she has reason to belief that this cat is not female. In interpreting these utterances,
there is thus no implicit conditionalization involved. Now the only difference between B’s utterance in this dialogue and the one in (14) is that the argument of the adverb is not a sentence anaphor, but a sentence. It is hard to see how this could affect the possibility for implicit conditionalization. One would expect that it is possible to read B’s utterance as stating that this cat is necessarily female provided that it is male. But such a reading seems completely unavailable. Conclusion: the anaphors so and that in (14) cannot have the consequent as their referent. As they can neither refer back to the entire sentence uttered by A, this exhausts the options of the Lewis/Kratzer analysis.

2.2 Gibbardian standoffs

A related problem crops up concerning Gibbardian standoffs. Gibbard (1981, 231) argued that conditionals have no truth values, see also Edgington (1995), Bennett (2003). His argument is based on the following example concerning a poker game between Sly Pete and Mr. Thomas Stone. Stone has bet up to the limit of the hand, and it is now up to Sly Pete to call or fold. Jack saw both the hand of Pete and that of his opponent, and saw that Pete has the losing hand. He thus believes that (19a) is true. Zack, on the other hand, saw the hand of Pete’s opponent and signaled it to Pete. He knows that Pete always plays to win, therefore he believes (19b):

(19) a. If Pete called, he lost.
    b. If Pete called, he won.
Gibbard argued that it cannot be the case that both conditionals are true (for the two conditionals are contradictory), neither can they both be false, nor is it the case that one of them is true but the other isn’t, for, which one should we prefer? Jack and Zack seems to be equally justified in making their claims.\(^7\) Gibbard concluded that, since no truth-value seems right for these sentences, indicative conditionals generally lack truth-values. Though I think this conclusion is too rash (I think conditionals only lack truth-values in case their antecedent is not true), I think the example is puzzling. What needs to be explained is (i) why Jack and Zack seem to be disagreeing, and (ii) why, at the same time, both have spoken correctly.

What does the Lewis/Kratzer theory have to say about these examples? In order to explain that neither Jack nor Zack has made an error, one might propose that (19a) is relative to the evidence available to Jack, while (19b) is relative to the evidence available to Zack.\(^8\) This suggests the following logical forms:

\(^7\)Some believe that Jack’s knowledge is better than Zack’s. However, more symmetric cases can easily be constructed, see Edgington (1995, 294).

\(^8\)This follows Kratzer’s (1986) remarks about epistemic modals:

Suppose a man is approaching both of us. You are standing over there. I am further away. I can only see the bare outlines of the man. In view of my evidence, the person approaching may be Fred. You know better. In view of your evidence, it cannot possibly be Fred, it must be Martin. If this is so, my utterance of (ia) and your utterance of (ib) are both true.

(i) a. The person approaching might be Fred.
    b. The person approaching cannot be Fred.

Had I uttered (ib) and you (ia), both our utterances would have been false.
However, this analysis has nothing to say about why Jack and Zack appear to be disagreeing. The forms in (20a) and (20b) do not contradict one another, so what is there to disagree about? What is more, the next dialogue between Jack and Zack seems perfectly felicitous:

(21) Jack: If Pete called, he lost.
Zack: That’s not true! If Pete called, he won.

We may again ask: what does that refer to? Certainly not to the proposition that for all Jack knows Pete lost if he called. Zack is not saying that Jack is wrong about his own mental state. Nor can we say that that targets the consequent of Jack’s conditional. Zack is not flat-out denying that Pete lost, he is denying that Pete lost, on the condition that he called. Instead, we want an bare conditional sentence to be available for anaphoric take-up. But in the Lewis/Kratzer analysis no such sentence is available.

The reader may have noticed that (21) is an instance of so-called faultless disagreement: Jack and Zack disagree with one another, yet both of them have spoken faultlessly. Faultless disagreement is thought to be the main argument in favor of relativism (Lasersohn 2005, Stephenson 2007a). Indeed, Stephenson (2007b), Weatherson (2009) argue for a relativist treatment of Gibbardian standoffs. The main idea is that the person on which the set of
relevant worlds depends is not a part of the Kaplanian context, but of the index of evaluation. Then this individual is not yet part of the proposition expressed, but only comes in when the truth value of this proposition is determined. Let's call this individual the judge. Then what Jack expressed is that as far as the judge knows, Pete lost if he called, while Zack expressed that as far as the judge knows, Pete won if he called. These propositions are contradictory, as no single judge would consider both of them true. At the same time, both propositions are true, albeit relative to different judges: what Jack said is true relative to Jack, and what Zack said is true relative to Zack.

It has been argued that faultless disagreement is not good enough to motivate relativism. Stojanovic (2007) argues that there are no truly convincing cases of the phenomenon, and that even if there were such cases, relativism would not provide a better analysis of faultless disagreement than its contenders. Whether there is a case to be made for relativism or not, for conditionals this move strikes me as unnecessary. As I will show below, all that is needed to account for Gibbardian standoffs are conditional connectives endowed with the right kind of meaning.

To sum up, there are at least two phenomena for which the Lewis/Kratzer approach fails to account: (i) the way conditionals can be picked up by anaphors in dialogues and (ii) the intuition that (19a) and (19b) are contradictory, yet neither Jack nor Zack was wrong to say what he said. The first problem strongly suggests that there are conditional connectives in the logical forms of natural language. To be sure, Gibbardian standoffs do not constitute a knock-down argument against the Lewis/Kratzer analysis, as
the analysis might be saved by moving to a relativist framework. On the truth-value gap semantics that I will propose in the next section, however, we get an account of Gibbardian standoffs for free.

3 Truth-value gap semantics

3.1 The meaning of if

In a footnote, Lewis already conceded that, besides denying the existence of conditional connectives, there is an alternative way of looking at domain restricting if-clauses:

What is the price of forcing the restriction-marking if to be a sentential connective after all? Exorbitant: it can be done if (1) we use a third truth value, (2) we adopt a far-fetched interpretation of the connective if, and (3) we impose an additional permanent restriction on the admissible cases. Let If Ψ, Φ have the same truth value as Φ if Ψ is true, and let it be third-valued if Ψ is false or third-valued. Let a case be admissible only if it makes the modified sentence either true or false, rather than third-valued. [...] A treatment along similar lines of if-clauses used to restrict ordinary, selective quantifiers may be found in Belnap (1970).

(Lewis 1975, 11, fn.1)

Lewis clearly didn’t think much of Belnap’s proposal, but I will argue that he was too quick to dismiss it. Still, it turns out to be somewhat of a challenge to figure out what Belnap’s proposal really is. The main idea is that only those
cases where the antecedent is true are relevant when evaluating a conditional. This is best illustrated by means of an example. Consider (22) and suppose that it concerns the next roll of an ordinary, six-sided dice (example by McDermott (1996)):

(22) If it is even, it will be a six.

Suppose that you had bet on (22). It seems clear that the bet is won when the result of the next roll is six, and lost when the result is four. But what if it is five? Intuitively, the bet is called off in this case. This is in line with people’s responses in the so-called truth table task, where subject are asked to rate the four truth table cases of a conditional. If one offers the options “true”, “false” and “irrelevant”, people will generally rate false antecedent cases as irrelevant (Evans et al. 1993). Clearly, this related to the so-called Ramsey Test: evaluating a conditional proceeds by first adding the antecedent to your stock of beliefs, and then considering the truth value of the consequent in the updated set of beliefs (Ramsey 1929).

Belnap initially proposes the following (quite vague) conditional semantics:

**Definition 3.1** (Belnap’s semantics)

If \( \varphi \) is true in \( w \), then what \( \varphi \rightarrow \psi \) asserts in \( w \) is what \( \psi \) asserts in \( w \). If \( \varphi \) is false or nonassertive in \( w \), then \( \varphi \rightarrow \psi \) is nonassertive in \( w \).

This definition follows Belnap (1970). In Belnap (1973), he proposes that \( \varphi \rightarrow \psi \) makes an assertion iff \( \varphi \) is not false, i.e. iff \( \varphi \) is either true or
nonassertive. But if the semantics is revised in this way, embedding Belnap’s conditional in the scope of a quantifier doesn’t (automatically) lead to domain restriction with the if-clause. For this reason, we opt for the older definition.\footnote{The newer definition is closer to Jeffrey (1963). Belnap suggests that Jeffrey purposely set up his semantics thus in order to preserve the validity of important logical laws like Contraposition. However, Jeffrey also needs to adopt a non-standard semantics for negation to get this right, so these laws do not come for free on the newer definition. Obviously, on the semantics in definition 3.1, we do not get Contraposition and other laws valid. But I will argue in section 4 that this is not a problem.}

As Belnap points out, the definition in 3.1 can be understood in two ways. First, it could be that conditionals with a non-true antecedent fail to express a proposition (let’s call this the interesting reading). Second, it could be that such a conditionals do express a proposition, but one that lacks a truth value (we’ll call this the boring reading). Somewhat surprisingly, Belnap claims that the second, boring reading doesn’t provide a satisfactory formalization of the meaning of conditionals (unfortunately, he does not qualify this remark). That is, he claims that in definition 3.1, the statement that what $\varphi \rightarrow \psi$ asserts in $w$ is identical to what $\psi$ asserts in $w$ “does not boringly mean an identity of truth-values but an identity of propositional content” (Belnap 1970, 4). In 3.1 $w$ is then a part of the context rather than an index of evaluation.\footnote{See also Bradley (2002) for a recent proponent of this view.}

However, I think that the interesting semantics is not at all what we want for conditionals. To see the problem, consider the following example by Edgington (1995, 289):

(23) If you press that switch, there will be an explosion.
Clearly, my saying (23) might well save your life, especially when the antecedent is false. But how is this possible if (23) fails to assert a proposition? How can (23) ever be used to persuade you to not press that switch, if my utterance of it fails to communicate something for you to grasp? Thus, the interesting semantics is plain absurd.\footnote{To be sure, Belnap seems to have seen the problem, writing that to say that a sentence A is nonassertive is “not to say that A is “meaningless” in any sensible sense; to say so would be a bad joke, for certainly it continues to have determinate semantic relations” (Belnap 1970, 2). Yet I don’t quite understand in which sense of meaning (23) has meaning in a context where it doesn’t express a proposition.}

The boring semantics seems to fare better in this respect. On this view, a conditional with a non-true antecedent does succeed in expressing a proposition, but this proposition lacks a truth value, or at least lacks one of the classic truth values \textit{true} and \textit{false}. We might formulate this as follows, where \# is the third truth value corresponding to undefinedness:\footnote{If \# stands for undefinedness, it doesn’t seem to matter much whether we choose the option that conditionals express \textit{total} functions from worlds to \{0, \#, 1\} and the option that they express propositions that are \textit{partial} functions, i.e. functions from worlds to \{0, 1\}. Strictly speaking, however, the two options need not amount to the same thing.}

\textbf{Definition 3.2} (Truth-value gap semantics)

\[
\begin{align*}
\llbracket \varphi \rightarrow \psi \rrbracket^{M,h}(w) &= 1, \text{iff}\ [\llbracket \psi \rrbracket^{M,h}(w) = 1 \text{ if } [\llbracket \varphi \rrbracket^{M,h}(w) = 1]; \text{ otherwise} \\
\llbracket \varphi \rightarrow \psi \rrbracket^{M,h}(w) &= \#
\end{align*}
\]

Thus, \(\varphi \rightarrow \psi\) only has a truth value in case the antecedent is true. And if the antecedent is true, the truth value of the entire conditional is the truth value of the consequent.

Given this definition, I have asserted something by uttering (23). What I asserted is neither true nor false, but at least I managed to get something
across: that I believe (23). Of course, that I believe (23) does not mean that I believe that it is true. Instead, what I have communicated is that I believe that (23) is true, given that it has a truth value, cf. (Edgington 1995, 290). If you take me to be reliable, you will also come to believe that (23) is true, provided that it has a truth value, i.e. that an explosion will occur on the supposition that you press that switch. Hence, you do not press it.

This account of the effect of uttering (23) presupposes a different norm for assertion than is normally assumed. It is in fact crucial that we adjust this norm now that we allow conditionals to be neither true nor false. If not, it seems that conditionals like (23) could never be felicitously uttered. In fact, Stalnaker (1975, 137, fn.2) saw this as a problem for an analysis of conditionals along the lines of definition 3.2. The traditional norm for assertion says that one should not make an assertion unless one has good reason to think that it is true. Clearly, (23) doesn’t meet this norm, for when I say this, I expect that my utterance will keep you from pressing that switch, and so I believe that (23) is truth value less. Why would anyone ever be willing to assert a proposition that has a very low probability of being true?

However, it seems reasonable that once we abandon bivalence, the norm for assertion shouldn’t be belief that the proposition expressed is true, but rather that the proposition expressed is true if it has a truth value. Thus, a conditional can be felicitously asserted, if its assertibility, defined as follows is high, cf. McDermott (1996):

**Definition 3.3** (Conditional assertability)
A conditional’s assertability is the probability that it is true, given that it has a truth value.

It is now clear how I could be prompted to assert (23).

### 3.2 Domain restriction

Inserting Belnap’s conditional in the scope of a quantifier leads to domain restriction of that quantifier provided that quantifiers are restricted to quantify over worlds in which their scope is defined. Entries for usually and must that correspond to this idea are (as is common, \( h[x/a] \) is that assignment function which assigns individual \( a \) to variable \( x \) and is otherwise the same as \( h \)):

**Definition 3.4 (Quantifiers)**

a. \[ [(\text{most } e: \varphi)(\psi)]^{\mathcal{M},h}(w) = 1 \text{ iff } \[\psi\]^{\mathcal{M},h[e/a]}(w) = 1 \text{ for most events } a \text{ for which } [\varphi]^{\mathcal{M},h[e/a]}(w) = 1 \text{ and for which } [\psi]^{\mathcal{M},h[e/a]}(w) = 0/1; \]
0 otherwise

b. \[ [(\text{all } w: \varphi)(\psi)]^{\mathcal{M},h}(w) = 1, \text{ iff } [\psi]^{\mathcal{M},h[w/a]}(w) = 1 \text{ for all worlds } a \text{ for which } [\varphi]^{\mathcal{M},h[w/a]}(w) = 1 \text{ and for which } [\psi]^{\mathcal{M},h[w/a]}(w) = 0/1; \]
0 otherwise

What does the ‘for which \([\psi]^w = 0/1\)’-part of the definition do? Intuitively, something of the form \((\text{most } e: \varphi)(\psi)\) says something about the proportions between two sets, i.e. the set of events for which \(\varphi\) is true and the set of events for which \(\psi\) is true. To evaluate such a statement, one has to consider
those events for which $\varphi$ and $\psi$ are true, and those for which $\varphi$ and $\psi$ are false. For some events, however, it may not be clear whether $\varphi$ and $\psi$ hold or do not hold. More precisely, for some events $a$, $[\psi]^{w,g[e/a]}$ may be $\#$. The above definition says that such events simply do not count, when evaluating $(\text{most } e: \varphi)(\psi)$. This seems to be exactly what we want. Counting those events among either the true instances or among the false messes up the predictions.

To see that the above definitions give us domain restriction, have a look at (24) and its representation. Within Belnap's system, sentences like (24) wear their logical forms on their sleeve: the entire conditional constitutes the scope of the quantifier:

(24) Harry usually drinks Butterbeer if he is happy.

$$(\text{most } e:)(\text{Harry is happy in } e \rightarrow \text{he drinks Butterbeer in } e)$$

This says that Harry drinks Butterbeer in most events in which the embedded conditional has a truth value, i.e. in most events in which Harry is happy.

We likewise predict that (25) is true iff Neville is the Chosen One in all (epistemically accessible) worlds in which Harry isn’t:

(25) If Harry isn’t the Chosen One, it must be Neville.

$$(\text{all } w: w \text{ is compatible with the evidence})(\text{Harry isn’t the Chosen One in } w \rightarrow \text{Neville is in } w)$$

We now see that it isn’t necessary at all to postulate that if is no genuine connective if we want to account for its domain restricting effect. We just
need to assign the right semantics to the conditional connective.\footnote{Now note that had we opted for the non-boring Belnap-semantics, we would have to modify the definitions of our quantifiers in a much more fundamental way in order to get domain restriction. For example, \textit{must} should then be restricted to contexts in which the conditional in its scope has an intension. But this implies that \textit{must} is a Kaplanian monster, for it should then shift the context to worlds where the conditional expresses a proposition. There is, however, little evidence that there is anything monstrous about quantifiers like \textit{must} and \textit{usually}. For instance, indexicals like \textit{I} do not shift their reference when embedded under these operators. I consider this an additional reason to opt for the boring version of Belnap’s semantics.}

Note that there is no syntax-semantics mismatch on this analysis. \textit{If}-clauses can, for the most part, simply be interpreted where they occur at surface. For instance, (24) can just be parsed as:

\begin{equation}
\text{(26) [usually [Harry drinks Butterbeer if he is happy]]}
\end{equation}

We need not assume that the \textit{if}-clause is a syntactic argument of the modal, nor that it ‘binds’ a free restrictor variable in order to explain domain restriction. Instead, the desired reading follows in the simplest way imaginable. Of course, for sentence-initial (restrictive) \textit{if}-clauses, as (26) we still face the problem that the surface order is the wrong order, so to speak. The problem vanishes by assuming that sentence-initial \textit{if}-clauses are related to sentence-final ones through movement. Then, at LF, the \textit{if}-clause must be reconstructed to the position where it was base-generated. This is in line with Bhatt and Pancheva (2006).

### 3.3 Bare conditionals

Given Belnap’s semantics, the following question immediately arises. What about bare conditionals like (27)?
(27) If Harry is not in the library, he is in the common room.

We actually have two options for analyzing this sentence. Given that we now have conditional connectives, one would expect that (27) is to be analyzed as (28a). In principle, however, we could also follow Lewis and Kratzer and stipulate that conditionals can only occur in the scope of quantifiers as in (28b). If so, there is a covert modal present in the representation for (27). But it seems clear that this is not the default analysis:

(28) a. Harry is not in the library → he is in the common room

b. (all w: w is compatible with the evidence)(Harry is not in the library in w → he is in the common room in w)

On the Lewis/Kratzer analysis, which has it that if is a meaningless device for domain restriction, it is obvious that a covert operator must be assumed that is responsible for the observed conditional meaning. But if we adopt Belnap’s semantics, it seems superfluous to resort to a covert operator, when our conditional has a meaning of its own. In fact, it is part of the attraction of the semantics that this is so. It would be very strange not to use this fact in the analysis of (27).

Yet, if (27) is to be treated as (28a), one wonders why conditionals are often felt to have an epistemic modal flavor. In other words, why does (27) appear to be equivalent to (29)?

(29) If Harry is not in the library, he must be in the common room.
The explanation runs as follows. By uttering (27), one expresses a proposition, which is a function from possible worlds to truth values, and the truth value may be #. Yet for indicative conditionals, the default is that there are some worlds in the common ground in which the conditional has the value 1. If not, that is, if it were given that Harry in fact is in the library, an utterance of (27) would not be felicitous. If this is indeed the normal use of indicatives (and I think it is fairly uncontroversial that this is so), the epistemic flavor is expected, for the epistemic alternatives to the actual world are exactly those worlds that are in the common ground (i.e. the worlds we consider live-options).

On empirical grounds, it is hard to choose between the analyses in (28a) and (28b). Theoretically, however, it is simplest to assume that the analysis in (28a) is right. If it is not possible for $\rightarrow$ to stand on its own, this has to be additionally stipulated, for it doesn’t follow from the semantics. One would have to come up with a motivation behind this restriction. In what follows, I will therefore assume that $\rightarrow$ may occur unembedded.

### 3.4 Conditionals in dialogue analyzed

Belnap’s semantics allows a straightforward treatment of the interpersonal traffic of conditionals in dialogues. As we saw above, the behavior of conditionals in dialogues suggests that there must be truly conditional forms to pick up for the anaphors so and that. In Belnap’s system, such a form is obviously provided. Our example can be analyzed as follows:

(30) A: If this cat isn’t male, it is female.
this cat isn’t male → it is female
‘if the conditional has a truth value, i.e. if this cat isn’t male, it is female’

B: Necessarily so.
(all w: w is compatible with the evidence)(this cat isn’t male in w → it is female in w)
‘in all worlds compatible with the evidence where the embedded conditional has a truth-value, i.e. where this cat isn’t male, it is female’

C: That’s very unlikely.
(few w: w is compatible with the evidence)(this cat isn’t male in w → it is female in w)
‘in few worlds compatible with the evidence where the embedded conditional has a truth value, i.e. where this cat isn’t male, it is female’

We see that the conditional form that the truth-value gap semantics assigns to A’ utterance can be plugged in under the modal operators in the replies by B and C. This gives us exactly the meanings we want.

3.5 Gibbardian standoffs analyzed

The problem concerning Gibbardian standoffs is how to explain that the following sentences are contradictory, while, at the same time, neither Jack

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14Simplifying things, I assume here that that’s unlikely can be analyzed analogously to few. Of course, this is not realistic, as the set of possible worlds is probably not countable, but nothing hinges on this.
nor Zack seems to have made an error:

(31) Jack: If Pete called, he lost.
Zack: If Pete called, he won.

Once we adopt the truth-value gap semantics, these sentences are naturally analyzed as conditional disagreements:

(32) a. Pete called → he lost
    ‘if the conditional has a truth value, i.e. if Pete called, then he lost’

b. Pete called → he won
    ‘if the conditional has a truth value, i.e. if Pete called, then he won’

Though the above sentences are strictly speaking not contradictory, they are on the assumption that the conditionals have truth-values. In this case, it is clear that if (32a) is true, then (32b) is false, while if (32a) is false, (32b) is true. I submit that this explains the intuition that Jack and Zack disagree. As I will also emphasize below in section 4, when we judge which semantic relations hold between sentences, we tend to tacitly assume that the sentences involved have truth-values.

What’s left is to explain why both Jack and Zack seem to be right in making their claims. This should be explained pragmatically. As said above, I take it that a speaker is justified in uttering a conditional if the probability that it is true, given that it has a truth value, is high. It seems reasonable
that speakers assess this probability relative to the evidence that is available to them. Since Jack and Zack are in different mental states, it should come as no surprise that they attach high probabilities to contradictory conditionals.

4 The logic of conditionals

By adopting a partial semantics for if, we loose the validity of certain laws which “warm the cockles of a logician’s heart”, as (Belnap 1973, 51) nicely puts it. In Belnap’s semantics, the following do no longer hold:15

(33) a. Contraposition:
\[ \varphi \to \psi \equiv \neg \psi \to \neg \varphi \]
b. Or-to-if-inference:
\[ \varphi \lor \psi \models \neg \varphi \to \psi \]

Any world in which \( \varphi \to \psi \) is true, is a world in which \( \psi \) is true, and therefore a world in which \( \neg \psi \to \neg \varphi \) lacks a truth value. It is easy to see that the reverse direction doesn’t hold either. Contraposition is thus ruled out. As for Or-to-if-inference, some worlds in which \( \varphi \lor \psi \) is true will make \( \varphi \) is true. These worlds will clearly not make \( \neg \varphi \to \psi \) true.1617

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15It is very likely that even more laws do no longer hold. We restrict attention to Contraposition and Or-to-if-inference, because the invalidity of these particular two is often used as an argument against non-classical theories, see for instance Lycan (2006).

16The reverse direction ‘If-to-or-inference’ of course does come out: any world in which \( \neg \varphi \to \psi \) is true, is a world in which \( \varphi \lor \psi \) is true in all worlds.

17All I am presuming here about the meaning of \( \neg \) and \( \lor \) is that \( \neg \psi \) is not true if \( \psi \) is, and that \( \varphi \lor \psi \) is true if \( \varphi \) is. I consider this uncontroversial. Yet the reader may wonder about the semantics of connectives other than \( \to \), now that we are working in a partial system. The semantics that Belnap assumes comes down to strong Kleene, except of course his definitions for \( \to \).
This is a problem, for Contraposition and Or-to-if-inference do seem to hold for natural language indicative conditionals, as (34) and (35) respectively show:

(34) If it is raining, we won’t play.
    Therefore, if we play, it isn’t raining.

(35) Either Oswald killed Kennedy, or someone else did.
    Therefore, if Oswald didn’t kill Kennedy, someone else did.

But if Contraposition and Or-to-if-inference are not valid, then why are (34) and (35) such compelling arguments? I submit that our judgments about the validity of (34) and (35) come about by the tacit assumption that the premise and conclusion have a truth value.

Strawson (1952) considers ways to make the inference from the Aristotellean A-form to the Aristotellean I-form valid:

(36) Every crow is black.
    Therefore, some crows are black.

Traditionally, the inference in (36) is not justified, for its premise is true in models in which there are no crows, yet its conclusion is clearly false in such a model. However, most English speakers find (36) valid.

Strawson sought to solve this puzzle by (i) abandoning the assumption that all sentences necessarily have a truth value, and (ii) redefining the notion of entailment. He assumes that A-forms are neither true nor false in case their subject term is empty. In addition, Strawson assumes that cases in which
the subject term is empty are irrelevant as far as entailment is concerned:

The rule that $A$ entails $I$ states that, if corresponding statements of these forms have truth values, then if the statement of the $A$ form is true, the statement of the $I$ form must be true; and so on.

(Strawson 1952, 177)

Let $|=_{S}$ be the kind of entailment that Strawson had in mind. This can be defined as follows:

**Definition 4.1** (Strawson-entailment)

$$\varphi |=_{S} \psi$$

iff

$$\varphi, \chi |= \psi \text{ (i.e. } \varphi, \chi \text{ classically entails } \psi \text{)}$$

where $\chi$ is a premise stating that the definedness conditions of all statements involved are satisfied

It is easy to see that (36) is Strawson-valid. The premise presupposes that there are crows. Strawson thought of this as a precondition for the premise to have a truth value: only if there are crows, can ‘Every crow is black’ be true or false. It follows that provided that the premise of (36) has a truth value, we are justified to conclude that some crows are black.

Belnap (1973) himself refers to this notion of entailment as a useful one in connection to his conditional semantics. Indeed, both Or-to-if-inference and Contraposition turn out to be Strawson-valid:

(37) a. Contraposition:

$$\varphi \rightarrow \psi \equiv_{S} \neg \psi \rightarrow \neg \varphi$$
b. Or-to-if-inference:
\[ \varphi \lor \psi \models S \neg \varphi \rightarrow \psi \]

Contraposition follows, i.e. \( \varphi \rightarrow \psi, \varphi \land \neg \psi \models \neg \psi \rightarrow \neg \varphi \) because there is no world which makes \( \varphi \rightarrow \psi \) and \( \neg \psi \) true. The same holds for the other direction. Clearly, Or-to-if-inference is also Strawson-valid: any worlds in which \( \varphi \lor \psi \) is true and in which \( \neg \varphi \rightarrow \psi \) has a truth value, is a world in which \( \neg \varphi \rightarrow \psi \) is true. Thus, assuming that the statements are either true or false, we get the inferences we want.

What does this mean for our inferences in (34) and (35)? In as far as these are valid, they are enthymematic inferences, i.e. inferences that rely on an additional tacit premise: that the statements involved have a (classical) truth value. It could well be that Strawson-entailment describes the way that human reasoning naturally works. Moreover, it seems that other linguistic phenomena are also sensitive to Strawson-entailment: von Fintel (1999) argues that NPI licensing is sensitive to Strawson-downward entailment.

5 Conclusion

To conclude, I want to emphasize that although truth value gaps have often been used to model presuppositions, the partiality we have written into the semantics of \( \textit{if} \) must be distinguished from presupposition. Thus, Bradley’s claim that “conditionals presuppose that their antecedents are true” (Bradley 2002, 354) is false. Someone who utters ‘All John’s children are bald’ in case John has no children, counts as having misled her audience. But this does not
hold for a speaker who uttered a conditional with a false antecedent. In fact, if conditionals presupposed their antecedent, one would expect that natural language didn’t contain any conditionals. On Gricean assumptions, if it were given that John has children, one shouldn’t say ‘If John has children, they are bald’, but just ‘His children are bald’.

Perhaps we should assume that the presupposition of conditionals is of the kind that is never already given, but that always has to be accommodated? This won’t work. Following Gazdar (1979), it is usually assumed that conditionals ‘If $\varphi, \psi$’ give rise to the clausal implicatures $\boxdot \varphi$ and $\boxdot \neg \varphi$, and that if a presupposition clashes with a clausal implicature, the implicature ‘wins’, i.e. the presupposition is canceled. It follows that if conditionals presupposed their antecedent, this presupposition would automatically be canceled. To sum up, Belnap-partiality must be concluded to have nothing to do with presupposition.

Given some presupposition theories, this is problematic. For instance, Heim’s (1983) context change potentials are essentially based on a partial semantics. If we were to combine this theory with our Belnap-semantics, we would thus be assuming two distinct kinds of partiality. But this seems impossible in as far as undefinedness comes down to a lack of semantic value; how can we distinguish between two non-existing values? On the other hand, other presupposition theories, most notably the anaphoric binding theory of

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18Ordinarily, that is. In some situations, for instance in an argument via modus ponens it is allowed to assert a conditional whose antecedent is already given.

19See also Stalnaker (1975) and van der Sandt (1988), though these authors do not work in Gazdar’s framework. For instance, in van der Sandt’s system, the presupposition is canceled because it clashes with the fact that the conditional was uttered. Of course, the underlying intuition is similar to Gazdar’s.
van der Sandt (1992), Geurts (1999), are fully independent of truth value gaps. Adopting Belnap’s semantics thus does not automatically commit us to there being different kinds of undefinedness. At any rate, it is clear that Belnap-gaps just are not presupposition-gaps.20

Summing up, I have shown that it is possible to reconcile the existence of domain restricting if-clauses with the assumption that if corresponds to a conditional connective. Not only is this possible, adopting conditional connectives in our logical forms has certain advantages. We need bare conditional sentences to be available for anaphoric take-up. We can give a straightforward account of Gibbardian standoffs. Last but not least, the truth-value gap semantics can be compositionally implemented in an easy way.

References


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20Note that the difference between presupposition and conditional assertion just alluded to provides another reason why we shouldn’t recruit Belnap’s semantics for all kinds of domain restriction. Quantifiers are normally felt to presuppose their domain (Strawson 1952, Geurts and van der Sandt 1999), precisely because uttering “Every crow is black” in case there are no crows is misleading. But analyzing this sentence in terms of \(\rightarrow\) would suggest that it is felicitous if there are no crows.


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